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Strategies and Equilibrium in a Game with a Moral Player

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Abstract: *The topic of morality has been investigated from the perspective of conditions and influences that result in immoral behavior or a deviation from moral or ethical choices. At the same time, moral perspective of the situation has not received such attention in behavioral economics. Previous research reveals that moral choice is not the best option in every situation, especially in experiments designed to provoke the deviation, but there is always a fraction of cases where individuals nevertheless choose the moral alternative. This paper entertains the proposition that such cases occur due to a skewed perception induced by individual's morality. Strategic implications are examined as a thought exercise within a normal form, simultaneous and repeated game framework, focusing on two elements of the skewed perception: moral satisfaction and a motivated ignorance of immoral alternatives. The findings point out to existence of game settings where a moral choice is the best strategy and leads to an equilibrium. However, any other game setting indicates unfavorable outcome for the moral player. In addition, skewed perception enhances perceived payoff whereby moral player suffers real loss before suffering perceived loss. Perception skewed by morality, therefore, causes strategic implications.*

Keywords: Morality; game theory; motivated ignorance; moral satisfaction; augmented game; strategies

Introduction

Some people choose to do good deeds regardless of their payoff. Consider volunteers, who clear the garbage from local environment or take care of abandoned pets. They might as well be a part of a zero-sum game, where the other player(s) continuously toss the garbage in the nature or abandon pets. Do their choices to do the moral thing have strategic implications? Do they consider doing it the other way or simply discard such possibility, deciding to stay ignorant to such options?

It is similar for everyday life or work-related situations. Many strategic interactions allow for strategies that can be labeled from morality perspective, starting with simple cooperative actions (help a colleague with his task, wear a protective mask and keep a distance, or borrow an umbrella) up to more serious framework and magnitude of consequences (choose a qualified employee instead of a friend, not to engage in corporate espionage or apply corporate social responsibility). While some moral choices work well when set on default (Gigerenzer, 2010), default behavior is at least questionable in strategic interactions. Nevertheless, a persons' morality level may guide the perception and interpretation of the strategic interaction. Given the morality level (e.g., high), the perception and interpretation of the strategic interaction may be skewed. Skewed perception and interpretation may occur due to unawareness or partial awareness in situations with strategic interaction (Halpern and Piermont, 2018; Halpern and Rego, 2007, 2008, 2009), regardless of the morality level.

However, in this special case, it is proposed that the role of morality is to govern the assessment of alternatives due to their moral satisfaction, as well as to motivate ignorance of the immoral alternatives. Moral approach to decision-making implies that some strategies and optimization paths are not going to be used by a moral individual, because of methods or the consequences of decisions (while there is also substantial evidence for deviation from such approach). Game theory framework enables examination of the outcomes and strategic implications. Thus, strategic implications of morality that arise from proposed motivated ignorance of immoral alternatives and moral satisfaction (in terms of payoff enhancement) will be examined as a thought exercise within a game theory framework.

The following section explores conceptual framework for the examination of the morality, followed by the examination of the strategies and equilibrium in the given settings, possibilities for the improvement of strategic position and the discussion and conclusion.

Considerations of morality as motivated ignorance about immoral alternatives

According to Vanberg and Congleton (1992), shifting a debate on morality and rationality into a game of normal form (prisoner-dilemma) framework enables an operational approach. Bowman (2008) argues that rational models' framework may be useful only if perfect information can be obtained and costs and benefits can be calculated. However, Curry et al. (2020) state that explanations of many different kinds of cooperation in game theory may relate to morality, and suggest that morality represents a combinatorial system, composed of differently combined elements for each individual. Also, each moral element has its immoral counterpart. The elements combine in a specifically designed *molecule*, shaping an individual behavior. Such approach is very close to behavioral, bounded rationality approach, allowing for individual differences to shape moral choices and behavior. The use of bounded rationality instead of classic approach to rationality, considers individuals rational in terms that they consider available strategies, outcomes and information in amount that they deem relevant (Hensher et al, 2005), thus allowing for the imperfections in a decision-making process, as well as influence of individual's values, preferences and intentions. Any situation requires individual's awareness, assessment and judgment, and in each part of the process a bias (in the broadest sense) may skew the individual's decision-making.

As Schwartz et al (2002, 1178) put it, "human beings routinely violate the principles of rational choice". In that sense, "bounded rationality research has been limited to exploring how people misuse and mis-integrate the information that is part of their cognitive set" (Bazerman and Sezer, 2016, 97). Chugh and Bazerman (2007) and Bazerman and Sezer (2016) introduce the bounded awareness approach into ethics, proposing a bounded ethicality. While bounded rationality approach examines different influences that cause the deviation from optimal outcome, bounded awareness focuses on the awareness of the alternatives, motivation and consequences.

The bounded awareness or unawareness may induce strategically important deviations, where individuals value objects of assessment differently, value them based on their preferences, or even introduce *virtual* moves outside of the formal game (Halpern and Piermont, 2018; Halpern and Rego, 2008). The combination of the two approaches may be interpreted in a way that individuals' morality shapes their awareness and assessment, meaning that the objects of assessment are perceived and evaluated through the prism of morality, causing the deviations in line with Halpern and Piermont (2018) and Halpern and Rego (2008) proposal.

Before the novel terminology, Gigerenzer (2010) suggested that the bases of a moral behavior are heuristics, meaning that the part of the information will be disregarded, and the outcome will not be derived using computation and optimization, but satisficing. Satisficing assumes that an individual can form a scale of satisfaction and can assign a certain level of satisfaction to each alternative or possible outcome. Thus, satisficing allows an individual to be aware of different alternatives and to *sort* them according to their own motivation, preferences, values, morality and intentions while achieving a certain level of satisfaction (usually, above the satisfaction threshold set by the same individual). Satisficing process may end as soon as the individual encounters the alternative that is good enough, meaning that is above the satisfaction threshold. However, the individual may discard previously chosen option if encounters a better one, thus approaching the maximization without formalizing it as a goal (Schwartz et al, 2002).

Motivated blindness, reasoning and ignorance refer to the situations where individuals fail to perceive, assess or interpret information that is not in line with their preferences or beliefs. Motivated blindness means that individuals *see what they want to see* or miss *contradictory information* (Bazerman and Tenbrunsel, 2011,5), when it suits them to remain ignorant. For example, if a moral person assigns satisfaction levels to alternatives, s/he might discard an immoral alternative without assigning any level of satisfaction, due to a choice to remain ignorant to that knowledge. But the individuals cannot believe anything they would like to, as they are at least partially restrained by the evidence and its possible interpretation (Gino et al, 2016).

While immoral alternatives might not necessarily produce enough motivation to be considered as satisfactory alternative, the possible loss or lack of satisfaction from the moral alternative, might be the evidence which partially restrains the moral person's choices. Moreover, while people hold on to the created self-image and feeling of morality, they might use the justification and rationalization which allow them to apply amoral or immoral strategies but retain the self-image of a moral person (Gino et al, 2016). Those concepts may be extended to moral ignorance (Mason, 2015).

While the usual approaches view the ignorance as a fallacy, McGoey (2012) suggests that the ignorance and *dismissing unsettling knowledge* may be a strategic approach in certain situations. While concept of motivated ignorance has been examined from the perspective of a deviation from the ethical choice, the generalization of such bias must also allow the opposite approach: an individual may not deviate from the ethical choice due to motivated ignorance. And such approach might also explain the moral minority in the conducted experiments on the topic, systematized by Gino et al (2016).

It can be noticed that stated approaches describe the same phenomenon, observed from different perspectives. Bounded ethicality, bounded awareness, motivated reasoning and satisficing share the common proposition of individual's deviation from optimal choices, which occurs due to imperfections of perception and reasoning process. A part of that process is awareness and assessment of the alternatives, and they can be skewed due to various factors. Motivation and preferences are among those factors, and they can be rooted in individual's morality. If that is the case, the moral alternatives lead to fulfillment and satisfaction. Thus, it is assumed that satisficing is guided by the underlying morality and that individual assigns certain level of satisfaction to each alternative. If alternatives have such qualities that relate to morality, the individual ranks the alternatives according to their perceived moral value (because higher moral value leads to the higher perceived fulfillment and obtained satisfaction). Hence, individual's payoff is governed by the moral satisfaction. Thus, a moral person faced with an immoral alternative should assign it a satisfaction level that sets it below the threshold.

Moreover, a moral individual might "choose" to ignore an immoral choice or some of its characteristics, such as higher objective payoff, assigning that choice a lower level of satisfaction. At the same time, higher level of moral satisfaction can be assigned to the moral choice, unrelated to objective gain. In this case, the moral satisfaction motivates the individual to ignore certain aspects of situation, which has the same effects on outcomes as bounded awareness and skewed interpretation of situation, which is in line with Halpern and Rego (2007, 2009) with the distinction in the source of the skewed interpretation: while the authors discuss unawareness/partial awareness in general, in this case the *unawareness* is motivated by morality.

For simplicity, moral satisfaction may be used as a measure of a difference between objective and perceived payoff in a small world, which enables analysis in a normal form game. While choices depend on endogenous players' factors, the game setting is exogenous (the perception of the game can change, but the game itself cannot). Thus, strategic implications of choices assessed by underlying morality criteria will be explored and examined as a thought exercise within a game theory framework.

Examination of the Strategies and Equilibrium

Single-stage Game - possible settings

To examine the moral player strategies, a single-stage game is proposed in the Table 1. The two players can choose between a moral *(M)* and immoral *(I)* strategy.

Table 1 Single-Stage Game of Complete Information

		Player 2	
		M	***I***
Player 1	***M***	*a, e*	*b. f*
	I	*c, g*	*d, h*

It is assumed that the Player 2 is indifferent to morality and that the players are not coordinated. The strategy space for Player 1 is $S_1 = \{M_1, I_1\}$, for Player 2 $S_2 = \{M_2, I_2\}$, and the set of possible strategies profiles is $S = \{(M, M), (M, I), (I, M), (I, I)\}$. In such setting, if a pure Nash equilibrium (Nash, 1950, 1951) exists, it is (s_1^*, s_2^*).

However, it is possible that the game involves mixed strategies, with strategy profiles $\sigma = (\sigma_1, \sigma_2)$, where probability distribution over strategies for Players 1 and 2 are $\sigma_1 = \{q_1^M, q_1^I\}$ and $\sigma_2 = \{p_2^M, p_2^I\}$, respectively. Assumed probabilities for Player 2 and Player 1 playing strategy *M* are *p* and *q*, respectively. Estimation of *p* helps Player 1 to determine the outcomes given the probable strategy of the Player 2. Similarly, the Player 2 wants to assess the probability that governs the choices of Player 1. If Player 1 were indifferent to morality, his/her actions would depend on the expected payoffs and probability related to Player 2 choice of strategy derived from indifference:

$$p = \frac{b-d}{(c-a+b-d)}, \qquad (1)$$

while Player 2's outcome is governed by probability related to Player 1 choice of strategy

$$q = \frac{g-h}{f-e+g-h}. \qquad (2)$$

and if $0 \le p \le 1$ and $0 \le q \le 1$ then (p, q) would lead to a Nash equilibrium.

If the observed game represents a zero-sum game (Morgensten and Von Neumann, 1953), there exists a clear motivation for deviation from the moral strategy, where $a = -e$, $b = -f$, $c = -g$, $d = -h$, and following standard representation of the zero-sum game also must be valid that $a = -b$, $b = -d$, $d = -c$, $c = -a$ and $e = -f$, $f = -h$, $h = -g$, $g = -e$, (as in game setting represented in Appendix A), leading to probabilities of players' moral choices of $p = \frac{b+d}{a+b+c+d}$ and $q = \frac{c+d}{a+b+c+d}$, where the higher absolute amounts of b and -c direct the probability toward moral strategy, and if $b = c$, then $p = q$.

For the purpose of further examination, it is allowed that the moral player perceives the game differently. If a moral player's perception of the game and own possible strategies is skewed by motivated ignorance (Bazerman and Tenbrunsel, 2011) induced by morality, that will lead to the creation of an augmented game. Such augmented game is either a subset of a real game or it may involve choices outside of equilibrium (similar to proposed models by Halpern and Piermont (2018) and Halpern and Rego (2008)).

If a moral player strictly sticks to morality in choice of a strategy, s/he should be indifferent to eventual loss caused by the 2nd Player's choice of immoral strategy, perceiving a as equal to b. Following morality presumption, the probability that the Player 1 will choose strategy M is $p = 1$, which also means that s/he ignores the second row of the payoff matrix in Table 1. This means that the Player 1 does not observe the game neither as a zero sum (given $a = b$), nor as a mixed strategy game (given that such probability distribution over underlying game strategies violates the premise of playing strategies with a non-zero probability).

For a strictly moral player, augmented perception of the game must be true

$$a \geq b \gg c > d. \qquad (3)$$

That means that the Player 1 ignores the difference between payoffs a and b for making a decision (also allowing him to cognitively perceive the difference, but to choose to ignore it), which means that s/he perceives payoffs a and b to be equal and to be moral. If a player chooses to play a certain strategy ignoring the differences in payoffs, s/he violates rational reasoning. The presented example of moral decision-making represents a violation of rational reasoning (in a classic sense) because of:

- Augmented perception of the payoff (and the game) as a result of motivated ignorance,
- Decision-making according to the rule in strategic interaction, rather than the strategy,
- Possible deviation from (unperceived) objective optimum.

This can describe the moral minority in the conducted experiments on the topic (Gino et al, 2016) to some extent. Nevertheless, the fact that there was a minority in conducted experiments shows that extreme deviations (from objectively higher gain, mostly in terms of monetary payoffs) caused by morality can rarely be observed. Besides that, most situations are not as simple, so the role of morality in the game requires further refinement.

The role of moral satisfaction

If the Player 1 plays the strategy M by default, then the game payoff will be determined by the choices of Player 2. The probability that the Player 2 plays a strategy M is $p(M)$. If the payoff f exceeds payoff e or the payoff h exceeds payoff g, it will diminish the probability that the Player 2 will play moral strategy. That leads to Player 2's choice of strategy I, and consequently a loss or diminished payoff for Player 1.

However, if a moral player perceives the payoff of the moral strategy as highest (as shown before), regardless to the objective payoff, it means that his perception of the payoff is enhanced, and not equal to real payoff. We suggest that the difference between the real and perceived payoff represents the moral satisfaction, denoted as *m*. As a moral player perceives a and b to be equal, then the strategic importance of moral satisfaction is revealed in the difference *m*, which takes the value of

$$m = |a - b|. \tag{4}$$

The difference in payoff denoted as moral satisfaction, *m*, enables the player to subjectively perceive the payoff as a gain, as long as s/he plays the moral strategy. That also means that moral player perceives the game as a positive sum game.

If the real payoffs match the perceived payoffs of the moral player, the game with a moral player can achieve a pure Nash equilibrium if and only if

$$e \geq g > f > h, \tag{5}$$

which is also the only situation where the moral choice of the Player 1 represents the best answer to the strategy of Player 2, with the outcome $s = (M, M)$.

This is true for different game settings where moral strategy either maximizes the gain or minimizes the loss, but not for a zero sum game (as (3) and (5) and assumed payoff relationships for a zero-sum game cannot be true at the same time) and not for the mixed strategies game (as the probability that the moral player will choose immoral strategy is zero). Thus, there exists a possible set of games where moral strategy is the best answer to the opponent's choice and augmented perception of the moral player might not be revealed by the choices. However, the role of moral satisfaction in strategic implications for the outcome are best revealed in games with different settings (that is, for any game where the payoffs do not follow (3) and (5)).

If it is assumed that for the Player 1 is always true $a - (b + m) = 0$, while $e - g = 0$ does not have to be true for the Player 2, then the game outcome depends on the payoffs g and e, and the choice of the Player 2. If $f > e$ and $h > g$, the Player 2 will choose strategy I with higher probability; if $f < e$ and $h < g$ the Player 2 will play strategy M with higher probability, and if $f = e$, the Player 2 plays will play either strategy with the probability of 0.5 leading to underlying mixed strategies (p, q).

There is a single order of payoffs that ensures that a moral strategy is the best answer of the Player 1 to the strategy played by Player 2. Given the temptation of the possible higher payoff from playing an immoral strategy in all other payoff settings, the possibilities for the achieving subjective optimum from the 1st player's perspective regard to perception of the payoffs (enhancement of the payoffs achieved by playing moral strategies, or diminishing the payoffs achieved by playing immoral strategies) or change in decision-making rules/subjective beliefs.

The role of penalties

The game does not have to be such that moral choices lead to Nash equilibrium, and in real life, frequently it is not. Moral questions defined by the law introduce penalties or punishment, such that for the Player 1 remains true $a - b = 0$ and $a = b > c > d$, where penalties reduce the payment from immoral action, increasing the motivation for choice of moral strategy. The second player must perceive $e \sim g$ and $f = h$ as true. That can be achieved by introducing the punishment, *P*, that results in $e > f - P$ and $g > h - P$. The punishment is effective in terms of diminishing payoff only if sufficiently reduces the payoff from playing immoral strategy,

$$P > f - e \text{ and } P > h - g. \qquad (6)$$

In special cases where Player 2 is not only indifferent to morality and acting rational but achieves additional payoff from playing an immoral strategy in terms of satisfaction, *s*, the value of *P* should be increased for the amount of *s*.

However, many everyday strategic interactions do not have consequences for society and are not regulated by the law. Such situations require additional consideration.

Repeated Games

The conclusion about possible outcomes extends to the repeated game, where Nash equilibrium leads to best responses for both players in each stage of the game.

If a Player 2 is not aware of the 1st Player's tendency to moral strategies, s/he will choose to play mixed strategies. For simplicity, let us examine the implications in a zero-sum game. The mixed strategies by Player 2 will allow the preferable outcome for the Player 1 in approximately 50% of the repetitions. In a such underlying setting, the real cumulative payoff of Player 1 will approximate to 0, while the perceived cumulative payoff rises over the stages (due to the moral satisfaction).

In addition, moral satisfaction may also arise from the opponent's choice of moral strategy, thus dividing the m to the part arising from moral player's satisfaction from playing a moral choice (m'') and the moral satisfaction arising from the opponent's choice of the moral strategy (m'):

$$m = m'' + m' = m'' + \frac{m}{2} p(M_2), \quad (7)$$

where $p(M_2)$ denotes the probability that opponent plays a moral strategy, and at the end of the stage it can take only values 1 or 0, if the opponent chose a moral strategy or not, respectively. Hence, the real payoff is determined with the played strategies $\pi\ (M, I)$ and perceived payoff is $\pi\ (M', I)$, where M' denotes real plus perceived additional benefits of playing a moral strategy. If Player 2 applies mixed strategies, while Player 1 always plays the moral strategy, the cumulative payoff of Player 1 will rise over stages (Appendix A). Notice that the positive cumulative payoff does not motivate the moral player to reconsider his morality or moral satisfaction, as long as the other player chooses mixed strategies.

However, if the Player 2 learns through the game, s/he will reveal the consecutive moral choices of Player 1, thus start to play the *strategy I* - that leads her/him to the higher gain while never suffering the loss. Although the perceived payoff allows Player 1 to experience gain each time s/he chooses a moral strategy, the real cumulative payoff might be negative. That might cause that the Player 1 reconsiders his moral satisfaction.

Point of morality reconsideration in repeated game

It is reasonable to assume that the point of reconsideration of moral satisfaction for Player 1 occurs when

$$\sum \pi\ (M', I) = 0. \quad (8)$$

At that point, the cumulative perceived payoff is equal to zero, meaning that not even moral satisfaction can compensate further loss.
The parsed cumulative payoff of the Player 1 is defined as

$$\sum \pi\ (M', I) = kM_1 p(M_2) + kmp(M_2) - lM_1\, p(I_2) + lm'' p(I_2), \quad (9)$$

where k is the number of the stages where Player 2 chooses the moral strategy and given that choice, the probability that Player 2 plays a moral strategy $p(M_2)$ takes value of 1, else 0; l is the number of the stages where Player 2 chooses immoral strategy and given that choice, the probability that Player 2 plays an immoral strategy, $p(I_2)$ takes value of 1, else 0; for a single stage $p(M_2) + p(I_2) = 1$; $k + l = n$ is the number of stages in a game; M_1 denotes the real payoff of the Player 1 for playing a moral strategy M_1 (the same symbol is used here for simplicity, as moral strategy is played by default); m denotes moral satisfaction of the Player 1 obtained from played moral strategy by himself and by opponent, m'' denotes moral satisfaction of the Player 1 obtained from playing a moral strategy (as in (7)).

Let us assume that the moral satisfaction represents a fraction of the real payoff (hence, is not bigger than the real payoff): $m = \alpha M_1$ and $m'' = \beta M_1, \alpha > \beta, \ \alpha, \beta \in [0, 1]$. The fractions may vary over individuals, thus allowing for specific individual combinations in the spirit of Curry et al. (2020) suggestion for individual approach to morality.
Substituting m and m'' in (9) at the end of the stage and assuming (8), we get:

$$kM_1(1 + \alpha) = lM_1(1 - \beta) \qquad (10)$$

$$\frac{k}{l} = \frac{1-\beta}{(1+\alpha)} \qquad (11)$$

If, for example, *α=1,* and *β= α /2,* then *k/l=1/4*. Based on the assumptions and calculated ratio, for this situation, the perceived cumulative payoff of a moral player reaches zero if Player 2 plays immoral strategy in four out of five stages of the repeated game. That leads to the real payoff for the Player 1 in the amount of $-3M_1$ for a five-stage game (Appendix B). The example sets α to the highest level and its increase cannot be examined, but the increase in β leads to more tolerance of the opponent's immoral strategies and more stages before morality reconsideration. The players with α and β set to the lower values will have less tolerance to the loss and gain less satisfaction from moral choice (i.e., their cumulative payoff will reach zero with fewer opponents' choices of immoral strategy, Appendix C) and will reach the point of reconsideration after fewer stages (game repetitions).

That shows that the augmented perception allows a moral player to suffer substantial real loss before s/he suffers perceived loss. However, at the point when player's cumulative perceived payoff reaches zero, there is no reason which would support the same behavior of a moral player. At this point, the moral player can: exit the game (when $l = k\frac{(1+\alpha)}{(1-\beta)}$), reconsider his morality satisfaction, or even enforce protection from of any applicable rule or common law.

Improvement of the Strategic Position

If a moral player does not choose to exit the game, then s/he should reconsider his/her options. If a choice is based on the real payoff and the moral satisfaction, then a possibility that player will consider both his actions and his morality must be anticipated as a possibility. If a player can reconsider his own moral satisfaction (that is, choose not to ignore real payoffs) then moral satisfaction can be susceptible to change, thus changing the perceived payoff.

Reconsideration of Moral Satisfaction

If the moral satisfaction can be changed, the level of *m* may depend on:

- Initial moral satisfaction m_{a0} and moral satisfaction factor α
- Cumulative payoff (whence positive cumulative payoff reinforces the moral belief, while negative cumulative payoff discourages the player from using the moral strategy)

- The belief about the opponent type and played moves (it can be assumed that the moral player initiates the game with the moral actions and continues to do so up to the stage where the gain of the moral satisfactions in a stage ceases to legitimate the real loss, more precisely when cumulative payoff reaches zero).

The moral satisfaction may be subject to change and an individual might choose not to stay ignorant; thus, it is necessary to assume that different level of the moral satisfaction may occur at each repetition of the game. It is also assumed that the moral player remembers the outcome of the previous games, namely cumulative payoff. In addition, the cumulative payoff contains the intuition about the opponent's strategy choices, which allows Player 1 to learn from previous stages. Hence it is assumed that the influence on moral satisfaction factor at the end of the stage may be presented as:

$$\alpha_n = m_{\alpha n} + \frac{1}{n}\left(\sum \pi(u^+) - \sum \pi(u^-)\right), \begin{cases} \alpha_n = 1, \quad m_{\alpha n} + \frac{1}{n}\left(\sum \pi(u^+) - \sum \pi(u^-)\right) \geq 1 \\ -1 < \alpha_n < 1, \; 0 < m_{\alpha n} + \frac{1}{n}\left(\sum \pi(u^+) - \sum \pi(u^-)\right) < 1, \\ \alpha_n = -1, \quad m_{\alpha n} + \frac{1}{n}\left(\sum \pi(u^+) \sum \pi(u^-)\right) \leq -1 \end{cases} \qquad (12)$$

The equation holds the intuition of the influence of the cumulative payoff: positive cumulative payoff reinforces and increases the morality, while negative cumulative payoff diminishes morality. The initial moral satisfaction factor is determined by α, where is assumed that $a_n \in [-1, 1]$ and $m_{a0} \in [-1, 1]$, with probability of moral player's choice of or deriving satisfaction from immoral or amoral level is $p(a \leq 0) \approx 0$ and $p(m_{a0} \leq 0) \approx 0$, respectivelly. That also means that initial influence of morality level on moral satisfaction amount is $\alpha_0 = m_{\alpha 0}, \alpha > 0$. It is also assumed that at the beginning of the first stage is valid $m_{a0} = m_{\alpha 1}$, hence $\alpha_0 = m_{\alpha 1}$. For the simplicity, it is assumed that moral satisfaction in a stage equals the moral satisfaction factor from the previous stage, for any two stages. The relationship expressed in (12) can be observed as following: at the end of each stage, α is calculated; α influences moral satisfaction; at the beginning of the next stage, $m_{a(n+1)}$ takes the value of α_n calculated at the end of previous stage of repeated game.

The α decreases over the stages n if $\sum \pi(u^-)$ rises. If the initial influence on moral satisfaction is $\alpha_0 = m_{a0} = 1$, then α reaches zero when $\frac{1}{n}\left(\sum \pi(u^+) - \sum \pi(u^-)\right) = -1$. For example, if $M_1 = 1$, that means that the moral satisfaction factor reaches zero only if each of the previously repeated games resulted in loss for the moral player, which can be achieved already at the end of the first stage (Appendix D). Also, it can be observed that the 2nd Player's actions have a bigger weight in the first stages, which in case of immoral choices causes rapid decline of the α and in case of game continuation, it takes longer for a moral satisfaction to recover as well as the perceived cumulative payoff (reminding of a bad first impression). However, if the 2nd player started the game with moral choices, the first one is more tolerant to occasional immoral choice. This approach is more restrictive towards the tolerance of immoral choices of 2nd Player than previous approaches, due to the more prominent influence of the real cumulative payoff.

It can be noticed that the morality itself does not have to change. Even though the player does not perceive moral satisfaction anymore, that still does not mean that s/he changed own core attitudes and principles. However, both payoff and moral satisfaction determine the player's action. In such a situation, a moral player will likely exit the game. However, if a moral player is ready to enter a grey area, s/he might consider revising his/her strategy set.

Reconsideration of the Strategy Set

Suppose that the moral player realizes that further choices of the same strategy do not lead to any kind of gain, moreover they lead to loss. If a moral player does not choose to exit the game and does not want to discard own principles, then s/he requires additional options. Again, moral player's perception of the game might be different than the opponent's perception.

If the moral player wants to improve his own strategic position, s/he may do so by perceived expansion of own strategy set or perceived reduction of opponent's strategy set (not necessarily on the conscious level). So, instead of the reconsideration of moral satisfaction, the player engages into reconsideration of the strategy set.

Expansion of the Strategy Set

The strategies of the moral player were $S_1 = \{M_1, I_1\}$, where s/he initially chose to play M_1 with the probability of 1. After the consideration, s/he might choose the lower level of morality, or even amorality, allowing him/her to use strategies $S_1' = \{M_1, \dots, I_1\}$. However, the initial game does not provide option for choosing the strategies between the moral and immoral strategy. If the Player 1 augments the game by expanding the strategy set with an amoral strategy (as a justified immoral strategy), that consideration is the result of his perception of the possible strategies or the choice to ignore/ augment the reality (in a different way). Moreover, the augmented perception of the Player 1 allows the choice of amoral strategy which might be simply concealed/justified immoral strategy. Suppose that the expanded game looks like the one presented in Table 2.

Table 2 Expanded Strategy set of Single-Stage game of complete information

		Player 2	
		M	***I***
Player 1	***M***	*a, e*	*b. f*
	A	*i, k*	*j, l*
	I	*c, g*	*d, h*

If underlying game considers a set of all alternatives for a given situation, then it does not support the notion of an amoral strategy. Thus, let there be no difference between amoral and immoral strategy, other than augmented perception of the Player 1, then $i = c, j = d, k = g,$ *and* $l = h$, so that only M and I are rationalizable strategies in the underlying game, as before. In such situation, Player 2 might assume that the only option for the Player 1 is to play the moral strategy. However, (1) and (2) hold true for this situation, taking the equilibrium to the intercept of the strategies with highest probabilities (Appendix E).

The underlying game does not contain the amoral strategy and augmented game perceived from the perspective of Player 1 introduces a strategy that is not a part of the underlying game, but - at the same time - it is an interpretation of the immoral strategy that is a part of the underlying game. In a way, this is related to the Gino et al (2016) presumption of motivated blindness, where people tend to miss or misinterpret the information contrary to their self-image. While this setting allows the moral player to play the immoral strategy (concealed or justified as amoral) and achieve optimal outcome, it also reveals the failure of application of morality principle. As we focus on examination of the moral player, the examination of expansion of the strategy set reaches its limits for this option, as it ceases to consider a moral player.

Influence on the perceived payoffs for the other player

The Player 1 might also approach to the problem by influencing the payoffs of the Player 2. Moral player could achieve that by reducing other player's set of strategies or perceived set of strategies by enforcing penalty (any applicable rule or common law). In such situation, the game looks as presented in the Table 3.

Table 3 Reduced Strategy set of Single-Stage Game of Complete

	Player 2		
Player 1		***M***	***I***
	M	*a, e*	*b. f'*
	I	*c, g'*	*d, h'*

The payoffs from playing the immoral strategy for the Player 2 are significantly lower after the application of the new rule. That makes the immoral strategy less desirable, and it is true: $a = e >= g' > f' > h'$. Such game leads to the equilibrium, which is also a pure Nash equilibrium: both players choose moral strategies. Player 1 continues to play a moral strategy by his own choice, but the Player 2 choses the moral strategy because the new rule diminished the payoff from the immoral strategy.

The influence on the perceived payoffs for the second player may be also introduced by a threat, however, analysis of such approach would require signal game setting and as such exceeds the scope of this examination.

Discussion and Conclusion

This paper examines the strategic possibilities for a moral player in a normal form game setting. The moral player's perception of the game is augmented due to his morality. The paper introduces examination of possible perceptions of the augmented game, where a moral player (a) ignores an immoral strategy, (b) perceives payoffs differently due to moral satisfaction and (c) considers a strategy outside of the game settings. The examination shows that if the game satisfies certain conditions regarding the payoff, the moral strategy can be a best answer to the other player's choice, which is in line with McGoey (2012) suggestion. However, for every other game setting, especially in repeated games, such approach leaves the moral player vulnerable to the choices of the other player.

The moral satisfaction may enhance perceived payoff, allowing the moral player to suffer real loss before suffering perceived loss, thus revealing the strategic implications of the skewed perception, which is in line with previous research (Halpern and Rego, 2007, 2009). Moreover, while moral player continuously chooses a moral strategy, the other player may recognize the pattern, choose immoral strategy and achieve gain each time. If a cumulative perceived payoff for a moral player reaches zero and any further repetition leads to a greater - both real and perceived - loss, a moral player cannot justify his morality level within a given game. If moral player chooses not to exit the game, then s/he must reconsider moral satisfaction level, as well as strategic possibilities. Additional extension of the augmented game for the addition of an amoral strategy (which is justified immoral strategy) opens a possibility for a strategic improvement of the moral player's position, but also leads to the failure of application of morality principle, if observed from the underlying game. If it is applicable to the situation, a moral player might enforce protection through penalties for immoral strategies, thus preserving his morality.

However, that is not always possible in everyday life, as not all situations, where morality plays a role, are prescribed by the law (e.g., a person suffered a loss of three umbrellas due to "borrowing"). If we consider the volunteers from the introduction, they might not know who their opponents are and thus not be able to enforce protection. In such a situation, a threat is suggested as a possible strategy, but that remains for future investigation in the signal game framework. If a moral player does not want to discard his morality, in repeated situations where enforcement of protection/penalty is not feasible, best option for a moral player is to exit the game. If the player cannot or does not want to exit the game, s/he can either continue to accumulate loss, or enter a "gray" area (e.g., break bad).

While the setting could be easily adjusted for examination of different principle or phenomenon with similar effects, the limitations follow from the assumptions of the examined situation. One of the limitations of the examined situation relates to the models' characteristics in general, as they tend to idealize, abstract, and caricature the observed situation. In that sense, the simplification of everyday situations with moral implications is inevitable. However, the examined moral satisfaction as payoff component (presumably derived by satisficing process) takes into account the value that a choice has for an individual, in contrast to the strictly objective payoff (such as monetary or materialistic one). The moral satisfaction accounts for subjective moral preferences of an individual. Nevertheless, we consider only the moral satisfaction and not the other benefits. For example, if a volunteer walks a dog, we consider only a moral satisfaction from that choice, while a possible positive influence on health is not examined but may exist. When applying this model to such situation, the health issue is an unaccounted externality. With an adjustment, the model might account for satisfaction derived from any form of influence subjectively weighted as positive, such as health benefits from walking a dog, as long as the volunteer cares for the health benefits. Such extensions remain for further research. Second, the process of motivated reasoning has not been examined, but assumed based on previous research. That process results in moral satisfaction and motivated ignorance, which are used as input for examination. Third, only two consequences of morality are considered and they both regard to the perception of the strategies/payoffs. Fourth, a normal form game framework is used. Possibilities for research extension refer to examination of different approach to morality and different game settings, such as sequential or signal games. And lastly, the game considers only two players, while further development may involve more players or coalitions of players.

Additional possibilities for future research regard to extension of the players' characteristics, such as religious beliefs, age, culture, gender, etc., which further suggests for combination of methods or use of other types of scientific methods.

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Appendix A. Simulation of payoffs for $m = |a - b|$ and $m' = m'' = \frac{m}{2}$ in a repeated zero-sum game

Game setting:

	Player 2		
Player 1		***M***	***I***
	M	*1, -1*	*-1, 1*
	I	*-1, 1*	*1, -1*

S1.

Player 1 choice	r	Player 2 choice	Player 1 real payoff	Player 2 payoff	m''	m'	Player 1 perceived payoff	Player 1 cumulative real payoff	Player 1 cumulative perceived payoff
M	0,2199	M	1	-1	1	1	3	1	3
M	0,5270	I	-1	1	1	0	0	0	3
M	0,0476	M	1	-1	1	1	3	1	6
M	0,9980	I	-1	1	1	0	0	0	6
M	0,0299	M	1	-1	1	1	3	1	9
M	0,6375	I	-1	1	1	0	0	0	9
M	0,9196	I	-1	1	1	0	0	-1	9
M	0,4583	M	1	-1	1	1	3	0	12
M	0,6485	I	-1	1	1	0	0	-1	12

M	0,9650	I	-1	1	1	0	0	-2	12

S2.

Player 1 choice	r	Player 2 choice	Player 1 real payoff	Player 2 payoff	m"	m'	Player 1 perceived payoff	Player 1 cumulative real payoff	Player 1 cumulative perceived payoff
M	0,5848	I	-1	1	1	0	0	-1	0
M	0,2583	M	1	-1	1	1	3	0	3
M	0,9009	I	-1	1	1	0	0	-1	3
M	0,5300	I	-1	1	1	0	0	-2	3
M	0,0213	M	1	-1	1	1	3	-1	6
M	0,9917	I	-1	1	1	0	0	-2	6
M	0,7098	I	-1	1	1	0	0	-3	6
M	0,7537	I	-1	1	1	0	0	-4	6
M	0,4555	M	1	-1	1	1	3	-3	9
M	0,9143	I	-1	1	1	0	0	-4	9

S3.

Player 1 choice	Player 2 choice	Player 1 real payoff	Player 2 payoff	m"	m'	Player 1 perceived payoff	Player 1 cumulative real payoff	Player 1 cumulative perceived payoff
M	I	-1	1	1	0	0	-1	0
M	I	-1	1	1	0	0	-2	0
M	I	-1	1	1	0	0	-3	0
M	I	-1	1	1	0	0	-4	0
M	I	-1	1	1	0	0	-5	0
M	I	-1	1	1	0	0	-6	0
M	I	-1	1	1	0	0	-7	0
M	I	-1	1	1	0	0	-8	0
M	I	-1	1	1	0	0	-9	0
M	I	-1	1	1	0	0	-10	0

S4.

Player 1 choice	Player 2 choice	Player 1 real payoff	Player 2 payoff	m"=1/2 (a-b)	m'=1/2 (a-b)	Player 1 perceived payoff	Player 1 cumulative real payoff	Player 1 cumulative perceived payoff
M	I	-1	1	1	0	0	-1	0
M	M	1	-1	1	1	3	0	3
M	I	-1	1	1	0	0	-1	3
M	M	1	-1	1	1	3	0	6
M	I	-1	1	1	0	0	-1	6

M	M	1	-1	1	1	3	0	9
M	I	-1	1	1	0	0	-1	9
M	M	1	-1	1	1	3	0	12
M	I	-1	1	1	0	0	-1	12
M	M	1	-1	1	1	3	0	15

S5.

Player 1 choice	Player 2 choice	Player 1 real payoff	Player 2 payoff	m''=1/2 (a-b)	m'=1/2 (a-b)	Player 1 perceived payoff	Player 1 cumulative real payoff	Player 1 cumulative perceived payoff
M	M	1	-1	1	1	3	1	3
M	I	-1	1	1	0	0	0	3
M	M	1	-1	1	1	3	1	6
M	I	-1	1	1	0	0	0	6
M	M	1	-1	1	1	3	1	9
M	I	-1	1	1	0	0	0	9
M	M	1	-1	1	1	3	1	12
M	I	-1	1	1	0	0	0	12
M	M	1	-1	1	1	3	1	15
M	I	-1	1	1	0	0	0	15

Note: n=10; r stands for randomly generated numbers, $r \in [0, 1\rangle$; in the simulations S1 and S2, Player 2 chooses mixed strategies based on the value of random number (for r<0,5 chooses M, else I).

Appendix B. Simulation of payoffs for $m'' = \alpha M_1$ and $m' = \beta M_1,\ \alpha = 1,\ \beta = \frac{1}{2}, \frac{k}{l} = \frac{1}{4}$

S1.

Player 1 choice	Player 2 choice	Player 1 real payoff	Player 2 payoff	Player 1 perceived payoff	Player 1 cumulative real payoff	Player 1 cumulative perceived payoff
M	M	1	-1	2	1	2
M	I	-1	1	-0,5	0	1,5
M	I	-1	1	-0,5	-1	1
M	I	-1	1	-0,5	-2	0,5
M	I	-1	1	-0,5	-3	0

S2.

Player 1 choice	r	Player 2 choice	Player 1 real payoff	Player 2 payoff	Player 1 perceived payoff	Player 1 cumulative real payoff	Player 1 cumulative perceived payoff
M	0,0385	M	1	-1	2	1	2
M	0,1752	M	1	-1	2	2	4
M	0,6164	I	-1	1	-0,5	1	3,5
M	0,1071	M	1	-1	2	2	5,5

M	0,3054	M	1	-1	2	3	7,5
M	0,5599	I	-1	1	-0,5	2	7
M	0,1735	M	1	-1	2	3	9
M	0,3765	M	1	-1	2	4	11
M	0,7243	I	-1	1	-0,5	3	10,5
M	0,9762	I	-1	1	-0,5	2	10

S3.

Player 1 choice	Player 2 choice	Player 1 real payoff	Player 2 payoff	Player 1 perceived payoff	Player 1 cumulative real payoff	Player 1 cumulative perceived payoff
M	M	1	-1	2	1	2
M	I	-1	1	-0,5	0	1,5
M	M	1	-1	2	1	3,5
M	I	-1	1	-0,5	0	3
M	M	1	-1	2	1	5
M	I	-1	1	-0,5	0	4,5
M	M	1	-1	2	1	6,5
M	I	-1	1	-0,5	0	6
M	M	1	-1	2	1	8
M	I	-1	1	-0,5	0	7,5

S4.

Player 1 choice	Player 2 choice	Player 1 real payoff	Player 2 payoff	Player 1 perceived payoff	Player 1 cumulative real payoff	Player 1 cumulative perceived payoff
M	I	-1	1	-0,5	-1	-0,5
M	M	1	-1	2	0	1,5
M	I	-1	1	-0,5	-1	1
M	M	1	-1	2	0	3
M	I	-1	1	-0,5	-1	2,5
M	M	1	-1	2	0	4,5
M	I	-1	1	-0,5	-1	4
M	M	1	-1	2	0	6
M	I	-1	1	-0,5	-1	5,5
M	M	1	-1	2	0	7,5

Note: n=10, unless cumulative payoff reaches 0 in the earlier stages; r stands for randomly generated numbers, $r \in [0,1\rangle$; payoff matrix is the same as in game setting in Appendix A. In simulations S2 and S3 Player 2 chooses mixed strategies based on the value of random number (for r<0,5 chooses M, else I).

Appendix C. Simulation of payoffs with different levels of α and β and *k/l*

S1. $m'' = \alpha M_1$ and $m' = \beta M_1$; $\alpha = \frac{1}{2}$, $\beta = \frac{1}{4}$, $\frac{k}{l} = \frac{3}{2}$

Player 1 choice	Player 2 choice	Player 1 real payoff	Player 2 payoff	Player 1 perceived payoff	Player 1 cumulative real payoff	Player 1 cumulative perceived payoff
M	I	-1	1	-0,75	-1	-0,75
M	M	1	-1	1,5	0	0,75
M	I	-1	1	-0,75	-1	0

S2. $m'' = \alpha M_1$ and $m' = \beta M_1$; $\alpha = \frac{1}{2}$, $\beta = \frac{3}{4}$, $\frac{k}{l} = \frac{1}{6}$

Player 1 choice	Player 2 choice	Player 1 real payoff	Player 2 payoff	Player 1 perceived payoff	Player 1 cumulative real payoff	Player 1 cumulative perceived payoff
M	M	1	-1	1,5	1	1,5
M	I	-1	1	-0,25	0	1,25
M	I	-1	1	-0,25	-1	1
M	I	-1	1	-0,25	-2	0,75
M	I	-1	1	-0,25	-3	0,5
M	I	-1	1	-0,25	-4	0,25
M	I	-1	1	-0,25	-5	0

Note: random numbers not shown for brevity; payoff matrix is the same as in game setting in Appendix A.

Appendix D. Simulation of moral satisfaction and moral satisfaction factor influence on perceived payoff

S1.

Player 1 choice	Random number generation	Player 2 choice	Player 1 real payoff	Player 2 payoff	α	$m_{\alpha n}$ $m_0 = 1$	Player 1 cumulative real payoff	Player 1 perceived payoff	Player 1 cumulative perceived payoff
M	0,3988	M	1	-1	1	1	1	2	2
M	0,3482	M	1	-1	1	1	2	2	4
M	0,9293	I	-1	1	1	1	1	0	4
M	0,3287	M	1	-1	1	1	2	2	6
M	0,5596	I	-1	1	1	1	1	0	6
M	0,4296	M	1	-1	1	1	2	2	8
M	0,2439	M	1	-1	1	1	3	2	10
M	0,2829	M	1	-1	1	1	4	2	12
M	0,7834	I	-1	1	1	1	3	0	12
M	0,8160	I	-1	1	1	1	2	0	12

S2.

Player 1 choice	Random number generation	Player 2 choice	Player 1 real payoff	Player 2 payoff	α	$m_{\alpha n}$ $m_0 = 1$	Player 1 cumulative real payoff	Player 1 perceived payoff	Player 1 cumulative perceived payoff
M	0,3455	M	1	-1	1	1	1	2	2
M	0,5459	I	-1	1	1	1	0	0	2

M	0,5250	I	-1	1	0,6667	1	-1	0	2
M	0,6415	I	-1	1	0,1667	0,6667	-2	-0,3333	1,6667
M	0,2650	M	1	-1	-0,0333	0,1667	-1	1,1667	2,8333
M	0,2699	M	1	-1	-0,0333	-0,0333	0	0,9667	3,8000
M	0,5414	I	-1	1	-0,1762	-0,0333	-1	-1,0333	2,7667
M	0,1083	M	1	-1	-0,1762	-0,1762	0	0,8238	3,5905
M	0,5039	I	-1	1	-0,2873	-0,1762	-1	-1,1762	2,4143
M	0,8225	I	-1	1	-0,4873	-0,2873	-2	-1,2873	1,1270

S3.

Player 1 choice	Random number generation	Player 2 choice	Player 1 real payoff	Player 2 payoff	α	$m_{\alpha n}$ $m_0 = 1$	Player 1 cumulative real payoff	Player 1 perceived payoff	Player 1 cumulative perceived payoff
M	0,5588	I	-1	1	0	1	-1	0	0
M	0,8468	I	-1	1	-1	0	-2	-1	-1
M	0,2012	M	1	-1	-1	-1	-1	0	-1
M	0,3007	M	1	-1	-1	-1	0	0	-1
M	0,1626	M	1	-1	-0,8	-1	1	0	-1
M	0,2395	M	1	-1	-0,4667	-0,8	2	0,2	-0,8
M	0,3039	M	1	-1	-0,0381	-0,4667	3	0,5333	-0,2667
M	0,4603	M	1	-1	0,4619	-0,0381	4	0,9619	0,6952
M	0,0715	M	1	-1	1	0,4619	5	1,4619	2,1571
M	0,5621	I	-1	1	1	1	4	0	2,1571

Note: r stands for randomly generated numbers, $r \in [0, 1\rangle$; payoff matrix is the same as in game setting in Appendix A. Player 2 chooses mixed strategies based on the value of random number (for r<0,5 chooses M, else I).

Appendix E. Simulation of payoffs with different levels of m'' and m' and mixed strategies for both players

S1. $m'' = \frac{1}{2}, m' = \frac{1}{4}$

r	Player 1 choice	r	Player 2 choice	Player 1 real payoff	Player 2 payoff	m''	m'	Player 1 perceived payoff	Player 1 cumulative real payoff	Player 1 cumulative perceived payoff
0,3401	M	0,0074	M	1	-1	0,5	0,25	1,5	1	1,5
0,1309	M	0,9392	I	-1	1	0,5	0,25	-0,75	0	0,75
0,2545	M	0,3641	M	1	-1	0,5	0,25	1,5	1	2,25
0,7178	A	0,4893	M	-1	-1	0	0	-1	0	1,25
0,1861	M	0,8282	I	-1	1	0,5	0,25	-0,75	-1	0,5
0,1934	M	0,2008	M	1	-1	0,5	0,25	1,5	0	2
0,9378	A	0,6735	I	1	1	0	0	1	1	3
0,8787	A	0,6060	I	1	1	0	0	1	2	4
0,7726	A	0,8294	I	1	1	0	0	1	3	5

0,8437	A	0,9991	I	1	1	0	0	1	4	6

S2. $m'' = 1, m' = \frac{1}{2}$

r	Player 1 choice	r	Player 2 choice	Player 1 real payoff	Player 2 payoff	m"	m'	Player 1 perceive d payoff	Player 1 cumulative real payoff	Player 1 cumulative perceived payoff
0,4944	M	0,4963	M	1	-1	1	0,5	2	1	2
0,6831	A	0,1676	M	-1	1	0	0	-1	0	1
0,0390	M	0,2806	M	1	-1	1	0,5	2	1	3
0,0111	M	0,1547	M	1	-1	1	0,5	2	2	5
0,0710	M	0,7699	I	-1	1	0	0	-1	1	4
0,8102	A	0,4394	M	-1	1	0	0	-1	0	3
0,2309	M	0,7700	I	-1	1	0	0	-1	-1	2
0,4442	M	0,4426	M	1	-1	1	0,5	2	0	4
0,7294	A	0,4368	M	-1	1	0	0	-1	-1	3
0,6588	A	0,2967	M	-1	1	0	0	-1	-2	2

S3. $m'' = 1, m' = \frac{3}{4}$

r	Player 1 choice	r	Player 2 choice	Player 1 real payoff	Player 2 payoff	m"	m'	Player 1 perceived payoff	Player 1 cumulative real payoff	Player 1 cumulative perceived payoff
0,2455	M	0,5588	I	-1	1	0	0	-1	-1	-1
0,3132	M	0,8988	I	-1	1	0	0	-1	-2	-2
0,5358	A	0,6327	I	1	-1	1	0,75	1,75	-1	-0,25
0,1133	M	0,1809	M	1	-1	1	0,75	2	0	1,75
0,1581	M	0,4103	M	1	-1	1	0,75	2	1	3,75
0,2141	M	0,2306	M	1	-1	1	0,75	2	2	5,75
0,5577	A	0,0413	M	-1	1	0	0	-1	1	4,75
0,2395	M	0,7499	I	-1	1	0	0	-1	0	3,75
0,2081	M	0,1924	M	1	-1	1	0,75	2	1	5,75
0,1191	M	0,9222	I	-1	1	0	0	-1	0	4,75

Note: r stands for randomly generated numbers, $r \in [0, 1\rangle$; payoff matrix is the same as in game setting in Appendix A. Player 1 and Player 2 choose mixed strategies based on the value of random number (for r<0,5 chooses M, else I).

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