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Models for Ordered Categorical Variables: An Application

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Abstract: *Ordered logit and ordered probit models, commonly used ordered categorical variable models, are used when the dependent variable is categorical and ordered. The validity of the predictions of these models depends on the assumption of parallel slopes. These models can be used if the assumption of parallel slopes is met. When this assumption is not met, the generalized ordered logit/probit model, which is more flexible in terms of assumption, or the multinomial logit model can be used by ignoring the assumption. In this connection, an exemplary data set was taken, and first of all, the assumption of parallel slopes was investigated. These models were compared using log-likelihood, Akaike Information Criteria (AIC), and Bayes Information Criteria (BIC) statistics for the validity of the models. In this way, the lowest log-likelihood, AIC, and BIC values were found for the generalized logit model.*

Keywords: Ordered logit model; ordered probit model; generalized logit model; generalized probit model

Introduction

The classical Ordinary Least Squares (OLS) method is used when the dependent variable is continuous (i.e., interval or ratio). This method includes many assumptions such as the error term of the model being normally distributed, its variance being constant, and the absence of multicollinearity and autocorrelation among the variables. However, when the dependent variable is binary, ordinal or descriptive, the OLS method is no longer the best-unbiased estimator (Agresti, 2018). If the dependent variable has more than 2 $(1, 2, 3, \cdots, j)$ levels (categorical), ordered or multinomial models are applied. In ordered models, the answers should be in natural order while determining the levels of the dependent variable. For example, the level of education - primary (level 1), secondary (level 2), high (level 3), university or college (level 4) can consist of 4 categories. There is a hierarchy between the dependent variable levels in the Likert-type scales widely used in survey studies. Levels of a dependent variable in an ordinal structure are usually coded as $1, 2, 3, \cdots$. The coding style here does not indicate superiority between levels. In multinomial models, on the other hand, the dependent variable levels should be independent of each other. For example, transportation preferences (bus, tram, metro) are independent. Therefore, when the dependent variable is categorical, the distinction between these models is made by considering whether the dependent variable levels are independent of each other or in a hierarchical order. While the example of transportation preferences is in a nominal or unordered structure, Likert-type scale samples are in an ordered structure. Classical regression methods cannot be applied if the dependent variable is on a nominal or ordered scale because they do not meet the assumptions. In this case, ordered or multinomial models can be considered. The common feature of ordinal category scales is that they have an ordinal structure but not a true metric structure. Therefore, nonparametric methods are preferred since it is not easy to investigate their suitability for parametric distribution (Layton and Katsuura, 2001).

Whether the dependent variable is ordered or unordered is important in model selection. In ordered models, the values of the dependent variable are coded by giving them a sequence number. The fact that the coding is in an ordered structure does not indicate that they have numerical superiority over each other. It shows an equal interval between the levels of the categorical dependent variable. The proportional probability and the ordered logit models (OLM) are among the most well-known methods (Rawat, 2023). The ordered logit model is estimated using a maximum likelihood estimation, and the coefficients can be interpreted as the change in the log-odds of the response variable for a one-unit change in the independent variable, with other variables held constant (Fullerton and Xu, 2012).

If the values of the dependent variable are multicategorical and ordered in nature, the ordered probit model (OPM) is the most used after the ordered logit model (OLM). The reason why these two models are widely used is that they give similar results. At the same time, the assumption of parallel slopes is valid in the two models. However, in practice, the logit model is more commonly used than the probit model. The probit model and the logit model assume normally distributed and logistically distributed error terms, respectively (Timur and Akay 2017). While Odds Ratio (OR) values are used to interpret the coefficients in OLM, the cumulative normal distribution is used in OPM. When comparing models, using OR values in interpretation is more practical.

Estimation methods used for ordered categories generally assume that the estimated coefficients (β_i) of the independent variables (X_i) do not vary across categories. This is known as the parallel slopes' assumption. The basic assumption of OLM and OPM is the parallel slopes assumption. If this assumption is not met for the estimated model, it is recommended to use alternative models because the estimates obtained are not reliable.
One of the alternative methods that can be used when the parallel slopes assumption is not met is the multinomial logit model (MNL). However, in the MNL model, the ordered structure of the dependent variable (Y) is ignored. MNL model is based on the Independence

of Irrelevant Alternatives (IIA) assumption. In this model, the categories of the dependent variable are nominally included. Today, it is seen that the dependent variable is categorical in most of the studies conducted in social sciences, physical sciences, and health sciences. If there is a ranking among these categories, the appropriate method should be chosen for the analysis to be carried out (Johnston et al., 2020). One of the most common mistakes made by researchers is to apply multinomial logit analysis, forgetting or ignoring that the categorical dependent variable is ordered (Hosmer and Lemeshow, 2000). Another multiple preference model is the multinomial probit model (MNP). This model can be used when the number of options is low (Arı and Yıldız, 2014). This is because the multi-choice probit model consists of nested integrals, and problem-solving is more difficult than the MNL model.

Choosing a model suitable for the nominal structure for a model with a sequential dependent variable will cause losses in the prediction results. Conversely, applying a sequential model to a model with a non-sequential structure or a nominal dependent variable will also cause deviations in the estimates (Azimi et al., 2020). Therefore, a model was sought that both took the ordered structure into account and relaxed the rather rigid assumption of parallel slopes. Thus, generalized models of these more flexible models began to be used instead of OLM and OPM.

The Generalized Ordered Logit Model (GOLM) or non-proportional risk model is used when the parallel slope assumption is unmet (Butler and Moffitt, 1982; Kolog et al., 2023). The generalized ordered logit model takes into account the heterogeneous influence of the independent variables under different thresholds (Fox and Andersen, 2006). Another method, the Generalized Probit Model (GOPM), can also be preferred. GOLM and GOPM both consider the ordered nature of the dependent variable and are flexible to the assumption of parallel slopes (Hilbe, 2018). The ordered dependent variables and their assumptions are given in Table 1 (Johnston et al., 2020).

Table 1. Models with ordered dependent variables

Ordered Models	**Assumptions**
Ordered Logit Model (OLM)	Parallel slopes assumption is required.
Ordered Probit Model (OPM)	Parallel slopes assumption is required.
The Generalized Ordered Logit Model (GOLM)	Flexible against parallel slopes and IIA assumption.
The Generalized Ordered Probit Model (GOPM)	Flexible against parallel slopes and IIA assumption.

Models with ordered dependent variables are generally among the multiple-choice models. These models also belong to the family of generalized linear models. Independent variables can be categorical or continuous variables. The difference between OLM and OPM is due to the distribution of the error terms. In OLM, the error terms are logistically distributed, whereas in OPM, the error terms are assumed to be normally distributed. It isn't easy to choose as there is not much difference between the distributions (Long and Freese, 2001). If the dependent variable is in an ordered structure, first, it should be tested whether the assumption of parallel slopes is met. There are different tests developed for the parallel slopes assumption. Among these, the LR and Brant tests are the most preferred. The Brant test is more comprehensive as it calculates each variable separately. In cases where the parallel slopes assumption is not met, generalized logit and probit models, which consider the ordered structure and are more flexible, as we mentioned before, can be used. Another method is to ignore the ordered structure and choose the MNL model (Güneri et al., 2022).

Material and Method

Ordered Logit Model (OLM)

The ordered logit model is one of the first models that come to mind when the dependent variable levels have more than two categories. Likert scale is widely used in data obtained by applying surveys, especially in studies conducted in different fields. This scale follows an ordered structure.

Suppose the dependent variable is in a natural order. In this case, classical regression cannot be applied because the intervals between the categories in the ordinal structure are not equal. (Kolog et all., 2023). However, using MNL for this type of data means ignoring the sequential structure of the dependent variable and not being able to use all the information in the dependent variable (Hosmer and Lemeshow, 2000). For these reasons, OLM has been widely used to analyze ordered data (Hardin and Hilbe, 2018). OLM, which is an improved form of the binary logit model, can be expressed as follows:

$$y^*_{OLM} = x^T\beta + \varepsilon \tag{1}$$

Here y^* is the latent variable, β is the unknown parameters, and ε is the error term. Given that the dependent variable has J ordered categories, the τ cut-off point/threshold values are as follows:

$$y_i = \begin{cases} 0, & y^* \leq 0 \\ 1, & 0 < y^* \leq \tau_1 \\ 2, & \tau_1 < y^* \leq \tau_2 \quad i \in \{1,2,3,\cdots,N\} \\ \vdots & \vdots \\ J, & \tau_{j-1} < y^* \end{cases} \tag{2}$$

Here, different logit construction formats are used when comparing the categories of the dependent variable. However, cumulative logit models are widely used in OLM because they are easier to interpret. This model uses cumulative probabilities up to a threshold. Adjacent category logit and continuation ratio logit models are other logit models used. These three logit models differ depending on which categories are compared and how they are compared (Hosmer and Lemeshow, 2000; Barak, 2005).

Another name for the ordered logit model is the proportional risk model. However, the ordered logit model is more widely used. In OLM, cumulative logits have the assumption of parallel slopes or the assumption of proportional hazards. In cases where this assumption is not met, the generalized logit model (GOLM) is one of the models used.

Ordered Probit Model (OPM)

The ordered probit model is another method commonly used to analyze models with ordered dependent variables. This model is seen as an alternative to the logit model. The current form of OPM was first proposed by (McElvey and Zavoina, 1975). The ordered probit model describes processes of generating random data by taking a series of discrete ordered results (William, 2016). In social sciences, ordered data can be used in bond ratings (such as AAA, AA, A, BBB, and AA+, A-, C categories), consumer preference and satisfaction research. In healthcare, ordered data are also common in research on people's health status. For example, when investigating the effect of a drug on patients, the dependent variable can be determined to have 3 levels: low (no effect), medium (reduction in symptoms), and high (full effect). Independent variables may include age, gender, sugar, blood pressure, and chronic disease. The ordered probit model fits these data by preserving the order of the dependent variable's categories without making any assumptions about interval distances between categories.
In traditional OPM, the assumption of parallel slopes is investigated, as in OLM. If this assumption is not met, the generalized version of this model can be used. This model is called the generalized ordered probit model (GOPM). The general model for the ordered probit model is as follows:

$$y^*_{OPM} = x^T\beta + \varepsilon \tag{3}$$

Here, y^* indicates levels of the dependent variable for OPM, x is the vector of independent variables, and β is the coefficients to be estimated. The categories of the dependent variable y^* can be shown as follows (Brant, 1990):

$$y_i = \begin{cases} 0, & y^* \leq 0 \\ 1, & 0 < y^* \leq \mu_1 \\ 2, & \mu_1 < y^* \leq \mu_2 \quad i \in \{1,2,3,\cdots,N\} \\ \vdots & \vdots \\ J, & \mu_{j-1} < y^* \end{cases} \tag{4}$$

Here, the μ's are the unknown threshold values that separate the categories. In OPM, errors are assumed to be normally distributed. The model can be estimated using the Gauss-Hermite Quadrature approach developed by Butler and Moffitt R. (McElvey and Zavoina, 1975). The $j-1$ threshold parameter of the y^* latent variable is obtained. If there is a constant term in the model, $j-2$ threshold parameters are estimated. Since the first threshold parameter is zero, the threshold parameters are all positive. The probabilities of each ordered outcome are as follows (Williams, 2006):

$$\begin{gathered} P[y_i=0]=P[\,y^*\le 0]=\phi(\mu_0-x_i\beta) \\ P[y_i=1]=P[\,0<y^*\le\mu_1]=\phi(\mu_1-x_i\beta)-\phi(\mu_0-x_i\beta \\ P[y_i=2]=P\big[\,\mu_1<y^*\le\mu_{2,}\big]=\phi(\mu_2-x_i\beta)-\phi(\mu_1-x_i\beta) \\ \vdots \\ P[y_i=j]=\phi\big(\mu_j-x_i\beta\big)-\phi(\mu_{j-1}-x_i\beta) \end{gathered} \tag{5}$$

$j=m$ the general format for the highest category is reduced as follows:

$$P[y_i=m]=\phi(\mu_m-x_i\beta)-\phi(\mu_{m-1}-x_i\beta)=1-\phi(\mu_{m-1}-x_i\beta) \tag{6}$$

Maximum Likelihood Estimation (MLE) can be used to estimate this model. For the MLE function, Z_{ij} is defined as the indicator variable equal to 1 if $y_i=j$, equal to 0 otherwise. If $\phi_{ij}=\phi[\mu_j-x_i\beta]$ and $\phi_{i,j-1}=\phi[\mu_{j-1}-x_i\beta]$, then the log-likelihood function is given as follows (Williams, 20016):

$$\ln L=\sum_{i=1}^{N}\sum_{j=0}^{m}Z_{ij}\ln[\phi_{ij}-\phi_{i,j-1}] \tag{7}$$

Generalized Ordered Logit Model (GOLM)

GOLM considers the dependent variable's ordered nature and does not restrict the proportional Odds assumption. OLM, also known as the Proportional Odds Model (POM), is a popular analytical method for ordering the dependent variable. However, Generalized Ordered Logit/Partial Proportional Odds (PPO) models are often a superior alternative (Rawat, 2023).

This model differs from the standard ordered or proportional probability model in relaxing the proportional probability assumption. If the parallel slopes assumption is not met, the parameters estimated by OLM may not reflect the truth. GOLM was first proposed by (Holm, 2023). Since using multinomial analyses in cases where the parallelism assumption is not met causes loss of information, a model that provides flexibility in this assumption and considers the ordered structure has been developed.

In this model, the effect of the difference ratio of the independent variable to the dependent variable is not equal, and the β coefficient is estimated differently for each category of the dependent variable (Holm, 2023; Long, 1997). Thresholds (breakpoints) divide the dependent variable categories into two groups. The first (lowest) coefficient vector corresponds to the division of the dependent variable into sets $\{1\}$ and $\{2,3,4,\cdots,j\}$. The second coefficient vector divides the dependent variable into the sets $\{1,2\}$ and $\{3,4,5,\cdots,j\}$. The $(j-1)$th coefficient vector corresponds to the division of the dependent variable into the sets $\{1,2,3,\cdots,j-1\}$ and $\{j\}$. GOLM fits j-1 concurrent logistic regression models, where the responses for these models are defined by narrowing the dependent variable to the new binary dependent variables defined by the sections described above (Jackman, 2000). In GOLM, suppose that the dependent variable takes the values of $\{0,1,2,\cdots,j\}$. GOLM estimates a set of coefficients and a constant for each of the $j-1$ points at which the dependent variable can be bisected. The generalized ordered logit model is given as follows:

$$\ln\left(\frac{\Pr\{y \leq j|x\}}{\Pr\{y > j|x\}}\right) = \tau_j - x\beta_j, \qquad 1 \leq j < J \tag{8}$$

Here, j is the ordered categories of the dependent variable, x is the vector of independent variables, τ is the cut-off point (threshold), and β is the logit coefficients vector. The j index in the β parameter shows that the coefficient of each independent variable can be estimated differently according to the cut-off points. This model is less efficient than other models because it estimates more parameters (Ling et all, 2023; Amemiya, 1985):

$$P(y_j > j) = g(x\beta_j) = \frac{\exp(\alpha_j + x_i\beta_j)}{1 + \{\exp(\alpha_j + x_i\beta_j)\}}, \qquad j \in \{1, 2, 3, \cdots, j-1\} \tag{9}$$

j is the category number of the ordered dependent variable. Probabilities can be determined for each value of y from 1 to j. According to Equation 9 given above, for the dependent variable y, each probability value from 1 to j is calculated as follows:

$$\left.\begin{array}{c} P(y_i = 1) = 1 - g(x_i\beta_1) \\ P(y_i = j) = g(x_i\beta_{j-1}) - g(x_i\beta_j) \\ P(y_i = J) = g(x_i\beta_{J-1}) \end{array}\right\}, \qquad j \in \{2, \dots, j-1\} \qquad 10)$$

Some known models are special cases of GOLM. When the dependent variable category is $j = 2$, it is equivalent to the GOLM logistic regression model. When $j > 2$, GOLM equals a series of binary logit regressions combining the categories of the dependent variable. For example, assuming that the dependent variable consists of 4 categories, for $j = 1$, the 1st category is compared with the 2nd, 3rd, and 4th categories. For $j = 2$, categories 3 and 4 are compared against categories 1 and 2. For $j = 3$, categories 1, 2, and 3 are compared with category 4 (Amemiya, 1985).

In GOLM, since the transition probabilities from one level of the dependent variable to the other will differ, model estimation is made for each level of the dependent variable. Accordingly, when the dependent variable consists of 3 categories, it requires estimating 2 different models.

Generalized Ordered Probit Model (GOPM)

The ordered probit model shows a normal distribution, and the ordered logit model shows a logit distribution. The resulting estimates will be biased and inconsistent if the distributions are determined incorrectly. Another method that can reduce the impact of distribution misidentification is GOPM. The observed variable y_i contains only a finite number of ordered results. Accordingly, the model is as follows (Liao, 1994):

$$y^* = x_i\beta + \varepsilon_i, \qquad 1 \le i < N \tag{11}$$

$$y_i = \begin{cases} 1, & {y_i}^* \le \alpha_1 \\ 2, & \alpha_1 < {y_i}^* \le \alpha_2 \\ 3, & \alpha_2 < {y_i}^* \le \alpha_3 \quad i \in \{1, 2, \dots, N\} \\ \vdots & \vdots \\ J, & \alpha_{j-1} < {y_i}^* \end{cases} \tag{12}$$

Here J is the number of mutually exclusive categories of y_i. For $1 < i < j$, the probability of observing a particular outcome is given by:

$$\begin{aligned} P(y_i = jIx_i) &= \mathrm{P}(\alpha_{j-1} \le y_i^* \le \alpha_j \\ &= \mathrm{P}\left(\alpha_{j-1} - x_i\beta \le \varepsilon_i \le \alpha_j - x_i\beta\right) \\ &= F\left(\alpha_j - x_i\beta; \theta\right) - F(\alpha_{j-1} - x_i\beta; \theta) \end{aligned} \tag{13}$$

Here F is the cumulative distribution function for ε_i, $\alpha_0 = \alpha_{1-1} = -\infty$ and $\alpha_J = \infty$. The presence of F leads to a maximum likelihood estimation. z_{ij} is defined as:

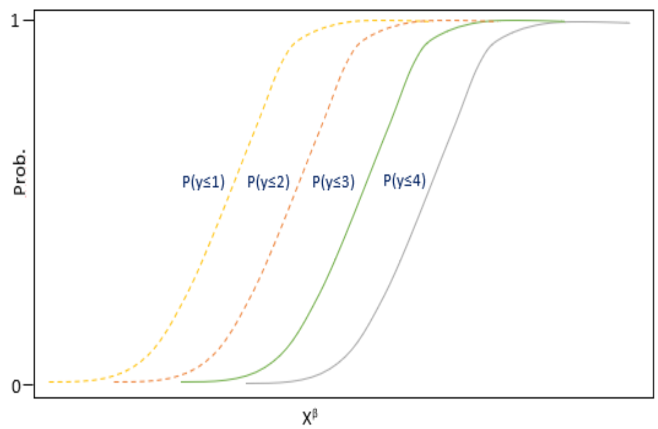
$$z_{ij} = \begin{cases} 1, & z_{ij} = j \\ 0, & other \end{cases} \tag{14}$$

From here, the log-likelihood function can be written as:

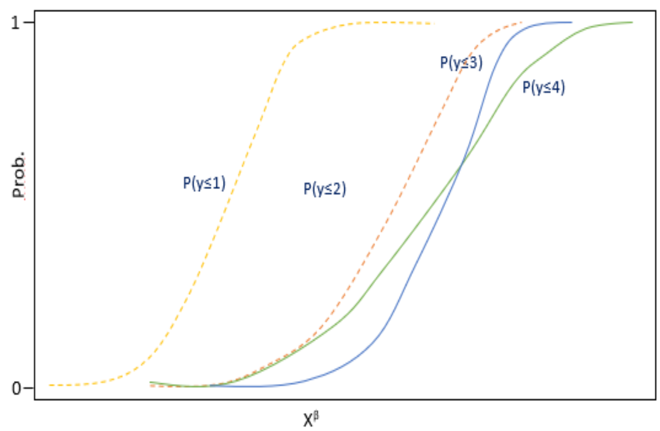
$$\log L = \sum_{i=1}^{N} \sum_{j=1}^{J} z_{ij} \log\left[F\left(\alpha_j - x_i\beta; \theta\right) - F(\alpha_{j-1} - x_i\beta; \theta)\right] \tag{15}$$

Parallel Slopes Assumption

The parallel slopes assumption is the most important assumption for ordered categorical data. According to this assumption, the categories of the dependent variable should be parallel to each other. In other words, parameter values should not change for different categories. If the parallel slopes assumption is met, the correlation between the dependent and independent variables does not change. Additionally, parameter estimates for cut-off points remain the same (Long, 1997). If the parallel slopes assumption is not met, applying the OLM and OPM models will cause erroneous interpretations. Figure 1 shows when the four categories' parallel slopes assumption is valid.



a. Parallel slopes assumption is met



b. Parallel slopes assumption is not met

Figure 1. Conditions in which the parallel slopes assumption is valid (a) and in which it is not (b)

When Figure 1 is examined, it is seen that the slopes for the four categories are the same in Figure 1a; that is, they are parallel, and they intersect in Figure 1b.

To investigate whether the parallel slopes assumption is met, the null hypothesis regarding the β coefficients can be written as follows:

$$H_0: \beta_{1j} = \beta_{2j} = \cdots = \beta_{(k-1)j} = \beta, \qquad j \in \{1, 2, 3, \cdots, J\} \tag{16}$$

Tests such as the likelihood ratio (LR) test and the Wald chi-square test are used to investigate whether the parallel slopes assumption is met (Barak, 2005; Fu, 1998). It also explores parallel slopes in tests such as Wolfe Gould.

One or more of these tests should be performed to investigate the parallel slope assumption for OLM and OPM models. Wald test statistic also examines the assumption of parallel slopes for each variable, while other methods investigate whether this assumption is met as a whole. If the assumption of parallel slopes is not met in ordered models, interpretations of the results for OLM and OPM will be incorrect. For this reason, alternative models such as generalized ordered logit and multinomial logit models can be used instead of ordered logit and ordered probit regression models to find correct results.

Brant Test

The Brant test tests the validity of the parallel slopes or proportional risk assumptions. This test tests the assumption of parallel slopes in general and gives information about which variable or variables cause this assumption to be violated. This test, which Brant first developed, is also known as the Wald test (Jackman, 2000). This test tests the equality of the estimated β_{j-1} coefficient of the $(j-1)$ binary logit model obtained from the dependent variable with j categories, and the equality of the coefficients estimated for each variable. The prediction results of the $(j-1)$ binary logit model are combined. The variance-covariance matrix of $\hat{\beta}^* = (\hat{\beta}_1', \hat{\beta}_2', \ldots, \hat{\beta}_{j-1}')'$ and $\hat{\beta}^*$ is constructed. The Wald test null hypothesis is as follows (Borooah, 2002):

$$\left.\begin{matrix} H_0: \beta_j = \beta \quad \text{or} \quad H_0: D\beta^* = 0 \\ H_1: \beta_j = \phi j \beta \quad \text{or} \quad H_1: D\beta^* \neq 0 \end{matrix}\right\}, \qquad j \in \{1, 2, 3, \cdots, k-1\} \tag{17}$$

Here, $(j-2)p \; x \; (j-1)p$ dimensional contrast matrix D is:

$$D = \begin{bmatrix} I & -I & 0 & \dots & 0 \\ I & 0 & -I & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ I & 0 & 0 & \dots & -I \end{bmatrix} \tag{18}$$

The Wald test statistic can be written using matrix D as follows:

$$W = \left(D\hat{\beta}^*\right)^T \left[D\hat{V}\hat{\beta}^*\left(\hat{\beta}^*\right)D^T\right]^{-1}\left(D\hat{\beta}^*\right) \tag{19}$$

The statistic asymptotically shows χ^2 distribution with the $(j-2)p$ degrees of freedom. For individual variables, the hypothesis is set as $H_0: \beta k_1 = \cdots = \beta_{k-1}$. Rows and columns corresponding to the coefficients to be tested can be selected from D, β^* and $\hat{V}\hat{\beta}^*$ matrices and tested with $(j-2)$ degrees of freedom (Fu, 1998). The Brant test suffers from two flaws common to many goodness-of-fit tests. First, if either j or p is greater, the test cannot be expected to be very powerful. Second, even if the test is powerful enough to detect deviations from proportionality, examination of the individual components of the test statistic may not provide a clear indication of the nature of the discrepancy detected (Borooah, 2002). Brant test is constructed as follows:

$$\delta_1 = 0;\ E(\tilde{\beta}_j) \approx \beta_1 + \delta_j \beta_1, \qquad j \in \{1, 2, \ldots, k-1\} \tag{20}$$

In this equation, $\widetilde{\beta}$ is a form of the nonlinear regression equation. The test of $\delta_j = 0$ is constructed by estimating the weighted regression of $\widetilde{\beta}$ on the inverse-weighted variance-covariance matrix of $\widetilde{\beta}$. Here, the test statistic is given as follows:

$$\chi^2 = +\hat{\delta}^T \hat{V}(\hat{\delta})^{-1} \hat{\delta} \tag{21}$$

Here, the variance-covariance matrix of $\hat{V}(\hat{\delta})\hat{\delta}$ shows the χ^2 distribution with $(k-2)$ degrees of freedom (Jackman, 2000).

Likelihood Ratio (LR) Test

The likelihood ratio (LR) test is used to investigate the effect of the independent variable or variables in the model on the dependent variable. The LR test is the most used test to investigate parallel slopes because it is available in most package programs. LR test is used to test the joint significance of the coefficients. In the model, where j is the number of categories, the null and alternative hypotheses are as follows:

$$\begin{gathered} H_0: \beta_1 = \beta_2 = \cdots = \beta_{J-1} = 0 \\ H_1: \text{at least one is different from } 0 \end{gathered} \tag{22}$$

In calculating the LR test statistic, the constrained and unconstrained models are estimated with the maximum likelihood method (ML), and maximum likelihood values are obtained. This test statistic, k being the number of independent variables, j being the number of categories of the dependent variable, conforms to the chi-square $\chi^2_{k(j-2)}$ distribution with $k(j-2)$ degrees of freedom (Ling et al., 2023). LR test statistic is obtained as follows:

$$LR = -2(L_0 - L_1) \tag{23}$$

In multiple-choice models, L_0 refers to the (constrained) model that meets the assumption of parallel slopes and L_1 refers to the model that does not meet the assumption of parallel slopes (unconstrained). For testing the parallel slopes assumption, the LR test tests the equality of the coefficients of all variables simultaneously. Accordingly, it cannot be determined whether the coefficients of some variables are identical versus binary equations while others are different. Therefore, Brant's Wald test is more useful as it tests the parallel regression assumption for each variable separately (Greene and Hensher, 2010). Failure to satisfy the parallel slopes assumption makes the ordered model results unreliable.

Application

The data used in this study were obtained from a survey to explore the factors influencing an individual's perception of the government's efforts to reduce poverty. It consists of the data in the 'carData' included in the R package and obtained from the World Values Surveys (WVS) for Australia, Norway, Sweden, and the United States. The poverty variable, which is the dependent variable and shows the perception of the individual, consists of 3 categories: about right, too little, and too much (for detailed information, see (Greene, 2019; Chen et al., 2023). The independent variables include an individual's religious belief, university degree, country, gender, and age. Explanations and descriptive statistics regarding the variables are presented in Table 2. The Stata 14 package program was used for analysis.

Table 2. Explanations of variables

Ordered Models	**Assumptions**
poverty	ordered dependent variable / too little, about right, and too much
religion	member of a religion/no or yes
degree	held a university degree / no or yes
country	Australia, Norway, Sweden, USA
gender	male or female
age	age (years)

When the independent variables are examined within the scope of the research, the variables other than age are categorical (Table 3). The levels of the dependent variable are given in Table 4. Accordingly, 1862 (34.60%) of 5831 individuals are in the about right category, 2708 (50.33%) are in the too little category, and 811 (15.07%) are in the too much category.

Table 3. Categorical independent variables

Variable	Categories	Freq.	Percent	Cum.
religion	no (1)	786	14.61	14.61
	yes (2)	4595	85.39	100.00
degree	no (1)	4238	78.76	78.76
	yes (2)	1143	21.24	100.00
country	Australia (1)	1874	34.83	34.83
	Norway (2)	1127	20.94	55.77
	Sweden (3)	1003	18.64	74.41
	USA (4)	1377	25.59	100.00
gender	female (1)	2725	50.64	50.64
	male (2)	2656	49.36	100.00

Table 4. Levels of the dependent variable

Poverty	Freq.	Percent	Cum.
about right (1)	1862	34.60	34.60
too little (2)	2708	50.33	84.93
too much (3)	811	15.07	100.00
total	5381	100.00	

Ordered Logit Model Results

OLM was applied first in this study since the dependent variable had three poverty levels. The dependent variable in the estimation of OLM consists of the categories about right, too little, and too much. OLM results are given in Table 5.

Table 5. Ordered logit model results

Poverty	Coef.	Std. Err.	z	p > z	Confidence Interval 95%	
					Lower	Upper
religion	-0.044	0.074	-0.600	0.551	-0.190	0.101
degree	-0.273	0.065	-4.210	0.000	-0.401	-0.146
country	0.191	0.023	8.380	0.000	0.146	0.235
gender	-0.112	0.052	-2.150	0.031	-0.215	-0.010
age	0.002	0.002	1.520	0.128	-0.001	0.005

/cut1	-0.680	0.196			-1.064		-0.297	
/cut2	1.716	0.197			1.329		2.103	
N = 5381	*LR chi2(5)*	*= 87.86*	*Prob > chi2*	*= 0.0000*	*Pseudo R2*	*= 0.0082*		

According to OLM, the degree, country, and gender variables were significant, while the religion and age variables were insignificant ($p < 0.05$). Significant variables can be interpreted according to the signs of the coefficients and also according to their odds values. Since the dependent variable has three levels, two cut-off points (threshold values) were obtained.

Cut-off points or thresholds used to distinguish between poverty levels are seen at the bottom (cut 1 and cut 2). If we show the ordered logit model as an equation, we obtain;

$$y^* = -0.044\, religion - 0.0273 \text{ degree} + 0{,}191 \text{ country} - 0.112 \text{ gender} + 0{,}002 \text{ age}$$

The representation of threshold values is as follows:

$$\text{y} = \begin{cases} 1 & -0.680 < y^* \leq 0.196, \\ 2 & 1.716 < y^* \leq 0.197 \end{cases}$$

According to the results obtained, when other variables are kept constant, the degree ratio reduces the probability of being at the 1-2-3 poverty level by 76% [OR= exp(-0.273)=0.76], while the country ratio reduces it by 1.21% [OR=exp(0.191)=1.21] and the gender ratio increases it by 89% [OR= exp(-0.112)=0.89]. $LR = 87.86$ chi-square value indicates that the model is significant for 5 degrees of freedom ($p = 0.000$). Necessary tests were carried out to check the parallel slopes assumption of the OLM.

Brant Test

Brant Test results are given in Table 6, and parallel slopes test results in Table 7. According to these results, the assumption of parallel slopes is unmet ($p = 0.000$).

Table 6. Brant test

	chi2	**p > chi2**	**df**
All	133.55	0.000	5
religion	0.43	0.514	1
degree	0.00	0.947	1
country	45.13	0.000	1

gender	7.14	0.008	1
age	69.06	0.000	1

The Brant test tests individual variables. According to the Brant test, the variables religion and degree satisfy the assumption of parallel slopes, but the other variables do not (Table 6).

Table 7. Parallel slopes assumption tests

	chi2	df	p > chi2
Wolfe Gould	131.6	5	0.000
Brant	133.6	5	0.000
score	132.5	5	0.000
Likelihood ratio	131.1	5	0.000
Wald	134.8	5	0.000

Different tests can be used to test the parallel slopes assumption. These tests test the presence or absence of the assumption as a whole. A trivial test statistic relative to these tests indicates that the final model does not violate the proportional ratios/parallel slopes assumption. An important test statistic shows that it violates the parallel slopes assumption. According to the results given in Table 7, the assumption of parallel slopes is not met according to the five methods.

Ordered Probit Model Results

OPM is another method that can be applied when the dependent variable is categorical. The results obtained for this model are given in Table 8.

Table 8. Ordered probit model results

Poverty	**Coef.**	**Std. Err.**	**z**	**p > z**	**Confidence Interval 95%**	
					Lower	Upper
religion	-0.028	0.044	-0.640	0.525	-0.114	0.058
degree	-0.163	0.038	-4.240	0.000	-0.239	-0.088
country	0.114	0.013	8.750	0.000	0.089	0.140
gender	-0.060	0.031	-1.950	0.051	-0.121	0.000

age	0.002	0.001	2.090	0.036	0.000	0.004
/cut1	-0.387	0.115			-0.613	-0.161
/cut2	1.060	0.116			0.833	1.288
N = 5381	*LR chi2(5)*	*= 95.96*	*Prob > chi2*	*= 0.000*	*Pseudo R2*	*= 0.008*

According to OPM, the religion and gender variables were insignificant. The other variables were found to be significant ($p < 0.05$). $LR = 95.96$ chi-square value indicates that the model is significant for 5 degrees of freedom ($p = 0.000$).

If we show the ordinal probit model as an equation, we obtain;

$$y^* = -0.028\, religion - 0.163 \text{ degree} + 0{,}114 \text{ country} - 0.060 \text{ gender} + 0{,}002 \text{ age}$$

The threshold values for this model are shown as follows:

$$\text{y} = \begin{cases} 1 & -0.387 < y^* \leq 0.115, \\ 2 & \;\; 1.060 \; < y^* \leq 0.116 \end{cases}$$

As with binary probit models, the results here are z scores. So the interpretation here is no different from binary probit models, except that the ordering is reflected in this case. For example, if we comment on the degree variable, it shows that a unit increase in degree will lead to a decrease of approximately 0.163 points in poverty in the z-score. Z-scores are similar in sign (as negative and positive variables). However, OPM is a model that requires the parallel slopes assumption. Therefore, the estimates are unreliable.

Generalized Ordered Logit Model Results

Since the parallel slopes assumption was not met, GOLM estimation results were obtained as an alternative method, and these results are given in Table 9.

Table 9. Generalized ordered logit model results

Poverty	**Coef.**	**Std. Err.**	**z**	**p > z**	**Confidence Interval 95%**	
					Lower	Upper
About Right						
religion	-0.016	0.082	-0.200	0.842	-0.177	0.144
degree	-0.274	0.071	-3.880	0.000	-0.412	-0.135
country	0.111	0.025	1.040	0.000	0.062	0.161

gender	-0.179	0.058	-3.090	0.002	-0.292	-0.065
age	-0.004	0.002	-2.250	0.025	-0.007	0.000
_cons	1.179	0.217	5.440	0.000	0.754	1.604
Too Little						
religion	-0.045	0.109	-0.410	0.683	-0.258	0.169
degree	-0.266	0.098	-2.700	0.007	-0.458	-0.073
country	0.340	0.032	10.480	0.000	0.277	0.404
gender	0.039	0.077	0.500	0.615	-0.112	0.190
age	0.016	0.002	7.120	0.000	0.011	0.020
_cons	-2.983	0.301	-9.920	0.000	-3.572	-2.394
N =5381	*LR chi2(10)*	*= 218.96*	*Prob > chi2*	*= 0.0000*	*Pseudo R2*	*= 0.0204*

As seen in Table 9, all the variables except religion were significant at the about-right level. At the too-little level, religion, and gender were found to be insignificant, while degree, country, and age were found to be significant ($p < 0.05$). The coefficients estimated in GOLM cannot be interpreted directly. For this reason, the difference ratios are calculated to interpret the model's coefficients.

Positive coefficients indicate that higher values on the explanatory variable make the participant more likely to be in a higher category y than the current one. Negative coefficients indicate that high values on the explanatory variable increase the probability of being in the current category or a lower category. $LR = 218.96$ chi-square value indicates that the model is significant for 10 degrees of freedom ($p = 0.000$).

At this stage, an analysis can be made using multinomial logit (MNL) regression to see how the coefficients differ when the information on the ordering of the categories is ignored. However, MNL models three levels of the poverty variable but does not consider that the order for the dependent variable is relevant.

Generalized Ordered Probit Model Results

Another method that can be used when the assumption of parallel slopes is not satisfied is GOPM. The results for this model are presented in Table 10.

Table 10. Generalized ordered probit model results

Poverty	**Coef.**	**Std. Err.**	**z**	**p > z**	**Confidence Interval 95%**	
					Lower	Upper
mleq1						
religion	-0.010	0.050	-0.200	0.841	-0.108	0.088
degree	-0.171	0.044	-3.930	0.000	-0.257	-0.086
country	0.066	0.016	4.250	0.000	0.036	0.097
gender	-0.110	0.035	-3.110	0.002	-0.179	-0.041
age	-0.002	0.001	-2.230	0.025	-0.004	0.000
_cons	0.739	0.132	5.570	0.000	0.479	0.998
mleq2						
religion	-0.032	0.060	-0.530	0.594	-0.150	0.086
degree	-0.161	0.054	-2.970	0.003	-0.267	-0.055
country	0.178	0.017	10.320	0.000	0.144	0.212
gender	0.017	0.042	0.400	0.691	-0.066	0.100
age	0.009	0.001	7.170	0.000	0.006	0.011
_cons	-1.659	0.162	-10.250	0.000	-1.976	-1.341

N = 5381 LR chi2(10) = 214.52 Prob > chi2 = 0.0000

As can be seen in Table 10, all the variables except 'religion' were found to be significant at the mleq1 level for GOPM. For mleq2 level, religion and gender were found to be insignificant, while degree, country, and age were found to be significant ($p < 0.05$). $LR = 214.52$ chi-square value indicates that the model is significant for 10 degrees of freedom ($p = 0.000$). These results are similar to the GOLM results in terms of the significance of the coefficients.

Conclusion

In most studies, especially in the field of social sciences and health, the dependent variable examined is categorical and ordinal. Therefore, model selection for models with ordinal categorical variables is an important issue. Among ordered models, classical OLM and OPM are widely preferred. However, one of the mistakes frequently made by researchers in studies is to neglect the ordinal structure of the categorical dependent variable and use the Multinomial Logit model.

OLM and OPM are preferred if the dependent variable is qualitative and ordered. However, these two models require the assumption of parallel slopes. The parallel slopes assumption assumes that the distance between each category of the outcome of these two models is proportional. In practice, it is seen that the assumption of parallel slopes is mostly not met, or researchers neglect this situation. The Wald test or the LR test, developed by Brant, are commonly used tests to test whether the proportional probabilities, that is, the parallel slopes assumption, are valid.

When the assumption of parallel slopes is unmet, GOLM and GOPM are preferred if the dependent variable has more than two categories and a natural ordering. It is recommended to apply MNL and MNP regression models if it does not have a natural ordering. When the parallel slopes assumption is violated, the MNL model is one of the most preferred models because it is easy to apply and interpret. However, for this model to be used, it must satisfy the Independence of Irrelevant Alternatives (IIA) assumption. Independence between alternatives is known as the relative probability of the two options not affecting the third option. This is the disadvantage of the MNL model. In addition, if there is an ordered dependent variable, the application of this model will not yield consistent results.

This study used survey data to investigate the factors affecting the individual's perception of governments' poverty reduction efforts for four countries. Since the dependent variable was measured with a three-category qualitative variable, comparisons were made by analyzing it with multiple preference models. Ordered models, which are widely used in the literature, were applied to the obtained data. Table 11 summarizes the results of these models.

Table 11. The goodness of fit statistics

	OLM	**OPM**	**GOLM**	**GOPM**
Log-likelihood				
Model	**-5326.26**	**-5322.21**	**-5260.707**	**-5262.929**
Intercept-only	-5370.19	-5370.19	-5370.188	
Chi-square				
Deviance (df=5374)	10652.51	10644.42	10521.414	
LR	87.863	95.957	218.962	214.52
p-value	0.000	0.000	0.000	0.000
R2				
McFadden	0.008	0.009	0.02	
McFadden (adjusted)	0.007	0.008	0.018	
McKelvey & Zavoina	0.018	0.022		

Cox-Snell/ML	0.016	0.018	0.04	
Cragg-Uhler/Nagelkerke	0.019	0.02	0.046	
Count	0.503	0.503	0.508	
Count (adjusted)	-0.001	-0.001	0.01	
IC				
AIC	**10666.51**	**10658.42**	**10545.414**	**10549.86**
AIC divided by N	1.982	1.981	1.96	
BIC	**10712.65**	**10704.55**	**10624.502**	**10628.95**
Variance of				
e	3.29	1		
y-star	3.352	1.023		

According to the results, the lowest Log-likelihood, Akaike Information Criteria (AIC), and Bayes Information Criteria (BIC) values were found for GOLM. The results obtained for OLM give the highest values. When the relevant literature is reviewed, it is seen that researchers generally prefer logit models. It is thought that the main reason for this is that the logit model is more widely known or is easier to interpret. However, as seen from the results, no significant difference was found between the two models.

As a result, before deciding which model to use, it should be checked whether the variables are in an ordered structure and whether the parallel slopes assumption is met. Otherwise, the results obtained may lead to erroneous estimates.

Author Contributions

All the authors equally contributed to this work. They all read and approved the final version of the paper.

Conflict of Interest

All the authors declare no conflict of interest.

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