JIS Journal of Interdisciplinary Sciences
Volume 9, Issue 1; 36-55
May, 2025. © The Author(s)
ISSN: 2594-3405

Is There a Place For *Entelecheia* in Modern Physics?

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Abstract: *While Aristotle coined two words, energeia and entelecheia in his description of change. The two concepts have ontological coverage and they may be used for any kind of particular change, including physical motion. Modern physics adopted energy as one of its basic concepts, built on a modified form of Aristotle's energeia. It is expected that entelecheia should have a counterpart in modern physics but few investigations were done. The teleological character of Aristotle's ontology makes difficult to build an exhaustive one-to-one correspondence between Aristotle's perception of motion and motion as we understand it now. However, such a correspondence may be easily emphasized in case of controlled motions, which are teleological by nature. We restrict the theory here to such motions and consider a simple mechanical system for which we identify a quantity, called entelechy, which obeys the two properties of Aristotle's entelecheia, i.e. (a) it is "holding its completion in itself" and (b) it converges towards kinetic energy, which is the modern equivalent of energeia. The shape of the system may be controlled in different ways, characterized by different entelechies and different amounts of work, in good agreement with common perception.*

Keywords: Aristotle; change; energeia; entelecheia; controlled motion

Introduction

Aristotle's system of thought was developed millennia ago and well prior to the development of modern physics, whose bases have been elaborated starting in the 17th century with the works of Galileo Galilei. The fundamental horizon of Aristotle's philosophical questioning is the problem of movement, and it is in the *Physics* that Aristotle most explicitly addresses this issue, with discussions on time, speed, *dunamis* and *energeia*, as pointed out by Martin Heidegger (see. e.g. Brogan, 2005). It is likely that that Aristotle's ideas somehow evolved into the ideas of modern physics but to fully demonstrate this would need detailed historical investigations showing an unbroken chain of connections leading from Aristotle's notions to the modern notions. Such a general approach is difficult. Here we simply observe that concepts developed originally based on metaphysical principles have had a very successful place in science after proper adaptation. A good example is the concept of *energeia* which remained accepted in physics.

Aristotle coined *energeia* and *entelecheia* in *Methaphysics* to describe change. Later on he used e*nergeia* in several books. Aristotle's most historically influential application of *entelecheia* outside of his *Metaphysics* is his claim in the second book of *De Anima* that the soul is the form or actuality of an organic body that makes it alive. Since Aristotle defined these concepts in relation with the genus of change it is expected that both *energeia* and *entelecheia* could be applied for any of its species, e.g. for physical movement. *Energeia* found, indeed, its way in modern physics as the root of the energy concept, starting with the works of Gottfried Wilhelm von Leibniz. Most modern researchers used the concept of *entelecheia* in a qualitative way only in case of animate matter. *Entelecheia* has not been used in relation with physical movement.

A recent article by Veldman (2024) pointed on the interaction between science and metaphysics and provided additional perspectives in the debate about the origins and the abandonment of physical teleology in the present-day interpretation of the Principle of Least Action. The present paper involves a formal approach which is similar with the way of using the Principle of Least Action in physics starting in late 19th and early 20th centuries, with the main idea being that the "final cause" characterizes the activity of theorizing, not the nature. This constitutes the so called "formal teleology" which allows a sort of formal unification of physics and some metaphysics concepts (Veldman 2024).

The teleological character of Aristotle's ontology makes difficult to build an exhaustive one-to-one correspondence between Aristotle's perception of motion and the motion as we understand it now. However, such a correspondence may be easier emphasized in case of controlled motions, which are teleological by nature.

Here we restrict to controlled motions with the aim to identify a quantity, called entelechy, which fulfils the features of Aristotle's *entelecheia*. In section 2 we review the meaning that Aristotle gives to *energeia* and *entelecheia* and how the concept of energy in modern physics relates to these terms. In section 3 we consider a very simple mechanical system for which we are able to define its energy and entelechy. We show that they converge each to another, following Aristotle's definition of *energeia* and *entelecheia*. Section 4 contains the conclusions.

Energeia* and *entelecheia

The nature of a thing is characterized by Aristotle by its form, shape or look, which was already present as a potential, an innate tendency to change, in that material before it achieved that form. All things have a proper kind of activity or work which, if achieved, would be their proper end. When things are most "fully at work" we can see more fully what kind of thing they really are (Sachs,1995, p.51, *Physics* 193b).

Trying to describe the change in ontology Aristotle defined the concepts of activity (*energeia*), fulfillment or actuality (*entelecheia*), and potency (*dunamis*) (Sentesy, 2020). When discussing the distinction between actuality and potentiality he used *entelecheia* (and *energeia*) in contrast with '*dunamis*'. Motion as a change is defined as the actuality (*entelecheia*) of a "potentiality as such" (Sachs, 1995, *Physics* 201a10-11, 201a27-29, 201b4-5). An interpretation of Aristotle's text is as follows (Sachs, 1995, pp 78-79): the being-at-work-staying-itself (*entelecheia*) is a genus with two species: motion and thinghood. Thinghood is the being-at-work-staying-itself of a potency (*dunamis*) as material. Motion is the being-at-work-staying-the-same of a potency as a potency.

Aristotle described the motivation behind the composition of the words *energeia* and *entelecheia* (Sachs, 1999, *Met.* IX.3 1047a30): "the phrase being-at-work, which is designed to converge in meaning with being-at-work-staying-complete" (Sachs, 1999). The modern dispute is mainly whether *energeia* means "actuality" or "activity" while a broad agreement is that *entelecheia* means "actuality" (Sentesy, 2020). There are opinions that *energeia* and *entelecheia* are interchangeable (Bradshaw, 2004; Durrant, 1993) but many researchers agree that the two words should be translated differently, taking into account Aristotle's argument that *energeia* converges with *entelecheia* (Sachs, 1999, *Met.* IX.8 1050a22–25).

An often used translation of the word *energeia* is "being-at-work' (Sachs, 2024), which solves several interpretation difficulties (Sentesy, 2020).

Aristotle uses a complex, carefully constructed term to expresses the precise meaning of *entelecheia*. This suggests that its meaning cannot be captured in a single word and its simple translation as "actuality" is not fully relevant (Sentesy, 2020). *Entelecheia* is a composite word based on *telos* and *echein*. *Echein* means both ongoing capability and activity, and being. The proper sense of *telos* is "excellence lack[ing] no part of the fullness it has by nature" (Sachs, 1999, *Met.* V.16 1021b22). Excellence here is not an end point in a sequence, but an ongoing virtuous potency (Sentesy, 2020). Aristotle argues against the idea that *telos* is an end point: what is primary is the ongoing condition of being *teleia* (Sachs, 2002, *Nicomachean Etics* I.10). The completion-related sense of *telos* is primary in respect with its sequence-related sense and Aristotle uses *telos* to mean *archē*: origin, a source of action, events, or being that directs or structures what arises from it. To be a *telos* is primarily to be that for the sake of which, which is different than (though not exclusive of) being an end point of change (Sachs, 1999, *Met.* IX.8 1050a6–8, XI.1 1059a35–37). *Telos* does not primarily mean "ended," or "finished" but "complete," "fully there," "whole," "entire" (Sentesy, 2020). Therefore, *entelecheia* should be rendered by "being-complete, or "staying-fulfilled," "holding onto completion," "holding itself in completion," "holding its completion in itself," "in active completion" (Sentesy, 2020). While e*nergeia* applies to individuals, *entelecheia* applies to composites, a broader class of things that includes individuals (Sentesy, 2020).

At Aristotle every thing has its own *entelecheia* and *energeia*. Therefore, there are so many *entelecheia* and *energeia* as many things are. Since Aristotle defined *energeia* and *entelecheia* to converge one another they must have related meanings. *Entelecheia* is a kind of completeness, whereas "the end and completion of any genuine being is its being-at-work" (*energeia*). The *entelecheia* is a continuous being-at-work (*energeia*) when something is doing its complete "work" (Sentesy, 2020). Sachs explains the convergence of *energeia* and *entelecheia* as follows (Sachs, 2024): “Just as *energeia* extends to *entelecheia* because it is the activity which makes a thing what it is, *entelecheia* extends to *energeia* because it is the end or perfection which has being only in, through, and during activity”.

Energy became part of modern physics in a form which modifies the *energeia* used by Aristotle in making the distinction between potentiality and actuality of a specific thing. Leibniz is credited to define for the first time what today is called energy. Leibniz explicitly acknowledged his debt to Aristotle whose doctrine of *entelecheia* (or ‘living force’) he regarded himself as restoring in a modified form (Klein, 1985; Sachs, 2024). By adopting a different approach than Aristotle, who stated that each thing has its own way of moving or changing, Leibniz said that instead, force, power, or motion itself could be transferred between things of different types, in such a way that there is a general conservation of this energy. The energy obeys its own laws of nature, whereas different types of things do not have their own separate laws of nature (Klein, 1985). The modern energy is a concept which applies to all things. Therefore, in present-day words energy may be seen as the equivalence class of the *energeia* of all things. Also, from Leibniz we derive our current notions of potential and kinetic energy, pointing to the actuality which is potential and the actuality which is motion, respectively, preserving the two characteristics in Aristotle's definition of motion.

Several comments on Leibniz’s approach are useful. Leibniz (1890) wrote: “...the entelechy of Aristotle, which has made so much noise, is nothing else but force or activity; that is, a state from which action naturally flows if nothing hinders it. But matter, primary and pure, taken without the souls or lives which are united to it, is purely passive; properly speaking also it is not a substance, but something incomplete”.

As seen, Leibniz identifies force or activity with entelecheia*. However,* entelecheia *is defined by Aristotle as a teleological, order giving element and how and why force or activity are teleological in nature, in terms of scope and shape, is not clearly explained by Leibniz. It seems that Leibniz’s force and activity are better related with* energeia*, the other of Aristotle’s concepts used to define motion, which is not teleological in nature, and this was the road followed by the next generations of researchers, starting with Thomas Young, William Thomson (Lord Kelvin) and William J. M. Rankine. Energy is a condition that describes the capacity to do work.*

Kinetic and potential energy are both related with Aristotle’s *energeia*. We shall show that, at least in the particular case of controlled motions, a quantity related with *entelecheia* may be also identified. Therefore, for such motions not only *energeia* but also *entelecheia* has a modern counterpart.

Simple example

The constraint adopted by scientists on metaphysics concepts is that they should have a consistent model or mathematical representation otherwise being disconfirmed (Veldman 2024). This constraint is adopted here when trying to finding a place in modern physics for *entelecheia*.

The concept of *entelecheia* is associated with the teleological character of motion in Aristotle's ontology. The teleological principle is not very often used in physics. Common physics problems involve known initial states while final states have to be found. The teleological principle involves known initial and final states. One of the areas of physics where the teleological approach may be adopted is the theory of controlled processes. *Entelecheia* is related to the final form of the thing and must be defined throughout the entire process of becoming, from the beginning until reaching the final state. It must therefore have character of potentiality but also must contain information regarding the final form. Shape is described in terms of positions of constituents. Knowing the shape of a body means knowing the positions of its constituents. Change of position is a matter of kinematics. Therefore, teleological problems involving shapes primary mean finding the kinematics of the constituents. The dynamics, even being necessary, may be thought to be secondary. On the other hand, due to their convergence, both *energeia* and *entelecheia* must have the same physical dimension. If we think in the usual terms of mechanics, a quantity related with *entelecheia* must be related to a sort of potential energy (which is different from actual energy, i.e. kinetic energy) but also be related to the final geometric configuration of the body.

We shall try to identify energy and entelechy, in the manner previously described, for a simple mechanical system. An inverse approach will be adopted. Starting from the initial and final states prescribed by the teleological principle we shall determine the kinematics able to connect them. Next, we shall find the dynamics supporting this kinematics, starting from the movement as it is to be and trying to find the force, say F_N, which makes this movement possible. Finally, we shall provide arguments that a sort of potential energy N defined as $F_N \equiv -\partial N / \partial x$ has the properties of *entelecheia*.

An example of simple problem involving a system with prescribed final shape is as follows. Assume a system of four particles in 2D rest positions, constituting the initial shape of the system. The rest position is convenient since shape is of primary interest for *entelecheia* while speed is just of secondary interest. Change in given time the particle system shape into a prescribed final shape (for instance, a given square). Find the particle kinematics and dynamics which make this shape change possible. This rather simple problem does not allow, however, a simple analytical solution.

Therefore, an even simpler 1D problem is considered in the following. Initially, at time $t = 0$ the system consists of a single particle of mass m in rest (velocity $v = 0$) at position $x = 0$

(initial system shape). After a given time $t = t_f$ the particle must be found in rest (velocity $v = 0$) at prescribed position $x = x_f$ (final shape of the system). Find the kinematics and dynamics of the particle.

This change of shape requires a force F_N controlling the particle movement. We shall obtain a sort of potential energy N from which F_N may be derived.

Since the velocity is zero at initial and final times, there is a time t', between 0 and t_f, for which the speed is a maximum, say v_{max}. For convenience we assume that the speed has just a single local maximum, which is also the global maximum. The position of the particle at time t' is denoted x'. The particle accelerates before time t' and decelerates after that time. Therefore, the controlled force F_N is acting along the axis x before position x' is reached and in contrary sense after this position.

The law of energy conservation between two states 1 and 2 is:

$$L_{12} = \Delta T\big|_1^2 \tag{1}$$

where $T = mv^2/2$ is kinetic energy while L_{12} is the work performed by the controlled force F_N between states 1 and 2. We denote $L_{0x'} \equiv \int_0^{x'} F_N dx$ and $L_{x'x_f} \equiv \int_{x'}^{x_f} |F_N| dx$ the work performed by the force F_N on the intervals $[0, x']$ and $(x', x_f]$, respectively. Equation (1) is used twice, for intervals $[0, x']$ and $(x', x_f]$, respectively. One sees that each of the work quantities $L_{0x'}$ and $L_{x'x_f}$ equal $mv_{max}^2/2$. The total amount of work L_{0x_f} performed by the force F_N on the interval $[0, x_f]$ is:

$$L_{0x_f} = L_{0x'} + L_{x'x_f} = mv_{max}^2 \tag{2}$$

Notice that the quantities $L_{0x'}$ and $L_{x'x_f}$ do not depend on the variation of the velocity v inside the intervals but only on the values of v at the ends intervals. Two cases of the time variation of speed, denoted Case 1 and Case 2, are considered next.

Variation of speed is linear in time (Case 1)

In Case 1, the movement is uniform accelerated/decelerated. Therefore, the variation of the speed is linear in time. The usual kinematics laws are:

$$v = a\Delta t \tag{3}$$

$$v^2 = 2a\Delta x \tag{4}$$

where a is the constant acceleration, with different values, say a' and $-a''\left(a''>0\right)$, before and after time t', respectively. We use the following notations: $\Delta x_{0x'} \equiv x'$, $\Delta x_{x'x_f} \equiv x_f - x'$, $\Delta t_{0t'} \equiv t'$, $\Delta t_{t't_f} \equiv t_f - t'$. The following relationships apply:

$$x_f = \Delta x_{0x'} + \Delta x_{x'x_f} \tag{5}$$

$$t_f = \Delta t_{0t'} + \Delta t_{t't_f} \tag{6}$$

Using Eqs. (3) and (4) twice, for the segments of space and time $\Delta x_{0x'}$, $\Delta x_{x'x_f}$ and $\Delta t_{0t'}$, $\Delta t_{t't_f}$, respectively, yields four equations, which together with Eqs. (5) and (6) give six equations for seven unknowns, namely $\Delta x_{0x'}$, $\Delta x_{x'x_f}$, $\Delta t_{0t'}$, $\Delta t_{t't_f}$, a', a'' and v_{max}. This system of equations is solved in terms of the acceleration a', which is used as a parameter. The results are:

$$v_{max} = \frac{2x_f}{t_t} \tag{7}$$

$$a'' = \frac{1}{\frac{t_f^2}{2x_f} - \frac{1}{a'}} \tag{8}$$

$$\Delta t_{0t'} = t' = \frac{v_{max}}{a'} \tag{9}$$

$$\Delta t_{t't_f} = \frac{v_{max}}{a''} \tag{10}$$

$$\Delta x_{0x'} = x' = \frac{v^2{}_{max}}{2a'} \tag{11}$$

$$\Delta x_{x'x_f} = \frac{v_{max}}{2a''} \tag{12}$$

Usage of Eqs. (7) and (2) gives the total amount of work L_{0x_f} performed by the force F_N on the space interval $\left[0,x_f\right]$:

$$L_{0,x_f}=\frac{4mx_f^2}{t_f^2} \tag{13}$$

Taking into account that the acceleration $a'\left(\geq 2x_f/t_f^2\right)$ is a parameter, we see that there are several strategies to obtain the given final shape of the system, all of them being associated with the same amount of work performed by the controlled force F_N.

The kinematics of the moment is obtained as follows. The time variation of the acceleration is:

$$a(t)=\begin{pmatrix} a' & (0\leq t\leq t') \\ -a'' & (t'<t\leq t_f) \end{pmatrix} \quad a''>0 \tag{14}$$

while the space variation of the acceleration is:

$$a(x)=\begin{pmatrix} a' & (0\leq x\leq x') \\ -a'' & (x'<x\leq x_f) \end{pmatrix} \tag{15}$$

The time variation of the speed is obtained by integration of Eq. (14) as follows:

$$v(t)=\int_0^t a(t)dt=\begin{cases} a't & (0\leq t\leq t') \\ a't'-a''(t-t') & (t'<t\leq t_f) \end{cases} \tag{16}$$

The space equation is obtained by integration of Eq. (16):

$$x(t)==\int_0^t v(t)dt=\begin{pmatrix} \dfrac{a't^2}{2} & (0\leq t\leq t') \\ \dfrac{a't'^2}{2}+a''\left(\dfrac{t^2}{2}-t't+\dfrac{t'^2}{2}\right) & (t'<t\leq t_f) \end{pmatrix} \tag{17}$$

The dynamics of the movement is obtained by using the Newton law:

$$ma(t)=F_N \tag{18}$$

Therefore, the time variation of the controlled force F_N is given by:

$$F_N(t)=ma(t)=m\begin{pmatrix} a' & (0\leq t\leq t') \\ -a'' & (t'<t\leq t_f) \end{pmatrix} \quad a''>0 \tag{19}$$

where Eqs. (14) and (18) have been used. The space variation of the controlled force F_N is obtained by using Eq. (15):

$$F_N(x)=ma(x)=m\begin{pmatrix} a' & (0\leq x\leq x') \\ -a'' & (x'<x\leq x_f) \end{pmatrix} \quad a''>0 \tag{20}$$

We assume that the controlled force F_N is related with a sort of controlled potential energy N through the following relation:

$$F_N(x) \equiv -\frac{\partial N(x)}{dx} \tag{21}$$

Therefore, in order to obtain N, there is a need for the space integration of F_N given by Eq. (21):

$$N(x) = -\int_0^x F_N dx \tag{22}$$

The result is

$$N(x) = -\int_0^x F_N dx = m\begin{pmatrix} -a' x & (0 \le x \le x') \\ -a' x' + a''(x - x') & (x' < x \le x_f) \end{pmatrix} \tag{23}$$

Here Eq. (20) has been also used.

Variation of speed is quadratic in time (Case 2)

In Case 2 a movement without acceleration discontinuities is considered. The following quadratic time variation of the speed is adopted:

$$v = p + qt + rt^2 \tag{24}$$

where p, q, r are constants.

First, the kinematics of the movement is treated. Equation (24) and the boundary conditions $v(t=0)=0$ and $v(t=t_f)=0$ give, respectively:

$$p = 0 \tag{25}$$

$$q + rt_f = 0 \tag{26}$$

The maximum speed v_{max} is reached at time $t = t'$. Therefore, from Eqs. (24) and (25) one finds:

$$\left.\frac{\partial v}{\partial t}\right|_{t=t'} = q + 2rt' = 0 \tag{27}$$

From Eqs. (26) and (27) one obtains the time when the speed is a maximum:

$$t' = \frac{t_f}{2} \tag{28}$$

From Eqs. (26), (24), (25) and (28) one finds:

$$v_{max} = \frac{s}{2} t_f^2 \quad (s \equiv -r > 0) \tag{29}$$

Usage of Eqs. (24), (25), (27) and (29) yields:

$$v = s\left(t_f t - t^2\right) \tag{30}$$

while the acceleration is given by:

$$a = \frac{\partial v}{\partial t} = s(t_f - 2t) \quad (31)$$

where Eq. (30) has been used. The constant s is obtained from $x = \int_0^t v(t)dt$ and $x(t = t_f) = x_f$, i.e:

$$x_f = \int_{t=0}^{t_f} v dt \quad (32)$$

giving:

$$s = \frac{6x_f}{t_f^3} \quad (33)$$

Finally, space, speed and acceleration are given by, respectively:

$$x = \frac{6x_f}{t_f^3}\left(\frac{t_f t^2}{2} - \frac{t^3}{3}\right) \quad (34)$$

$$v = \frac{6x_f}{t_f^3}\left(t_f t - t^2\right) \quad (35)$$

$$a = \frac{6x_f}{t_f^3}\left(t_f - 2t\right) \quad (36)$$

Usage of Eqs. (29) and (33) gives the value of the maximum speed:

$$v_{max} = \frac{3x_f}{t_f} \quad (37)$$

The work performed by the force F_N is obtained by using Eqs. (2) and (37):

$$L = \frac{9mx_f^2}{t_f^2} \quad (38)$$

The formal procedure of finding the dynamics in Case 2 requires obtaining $a(x)$ by eliminating t between Eqs. (34) si (36). Next, Eqs. (21) and (22) are used to obtain the controlled force F_N and the controlled potential energy N, respectively.

Discussion

Dimensionless symbols are defined in Table 1.

Table 1. Dimensionless symbols

Quantity	Symbol	Dimensionless symbol	Case 1	Case 2
Time	t	$\tau \equiv \frac{t}{t_f}$		
Space	x	$\xi \equiv \frac{x}{x_f}$		
Speed	v	$\nu \equiv \frac{v}{v_{max}}$	$v_{max} = \frac{2x_f}{t_f}$	$v_{max} = \frac{3x_f}{t_f}$
Acceleration	a	$\alpha \equiv \frac{a}{a(t=0)}$	$a(t=0) = a'$	$a(t=0) = \frac{6x_f}{t_f^2}$
Controlled potential energy	N	$\chi \equiv \frac{N}{a(t=0)x_f}$	$a(t=0) = a'$	$a(t=0) = \frac{6x_f}{t_f^2}$

The previous results are summarized in Table 2 in dimensionless form for both Case 1 and Case 2.

Table 2. Dimensionless equations. Dimensionless symbols are defined in Table 1

Equation number	Relationship
	Case 1, dimensionless parameter $\Lambda \equiv \frac{a' t_f^2}{2x_f} \geq 1$
1	$\xi = \begin{cases} \Lambda\tau^2 & \left(0 \leq \tau \leq \frac{1}{\Lambda}\right) \\ \frac{1}{\Lambda} + \frac{\Lambda}{\Lambda-1}\left(\tau^2 - \frac{2\tau}{\Lambda} + \frac{1}{\Lambda^2}\right) & \left(\frac{1}{\Lambda} < \tau \leq 1\right) \end{cases}$
2	$\nu = \begin{cases} \Lambda\tau & \left(0 \leq \tau \leq \frac{1}{\Lambda}\right) \\ \Lambda - \frac{1}{\Lambda-1}\left(\tau - \frac{1}{\Lambda}\right) & \left(\frac{1}{\Lambda} < \tau \leq 1\right) \end{cases}$

3	$\alpha = \begin{cases} 1 & \left(0 \le \tau \le \dfrac{1}{\Lambda}\right) \\ -\dfrac{1}{\Lambda - 1} & \left(\dfrac{1}{\Lambda} < \tau \le 1\right) \end{cases}$
4	$\chi = \begin{cases} -\xi & \left(0 \le \xi \le \dfrac{1}{\Lambda}\right) \\ -\dfrac{1}{\Lambda} + \dfrac{1}{\Lambda - 1}\left(\xi - \dfrac{1}{\Lambda}\right) & \left(\dfrac{1}{\Lambda} < \xi \le 1\right) \end{cases}$
	Case 2
5	$\xi = 6\left(\dfrac{\tau^2}{2} - \dfrac{\tau^3}{3}\right)$
6	$\nu = 2\left(\tau - \tau^2\right)$
7	$\alpha = 1 - 2\tau$
8	$\chi(\xi) = \int_0^{\xi} \alpha(\xi) d\xi$

Figure 1 shows the time variation of functions ξ, ν and α, for Cases 1 and 2. The moment of the discontinuity of the acceleration α and the change of slope of the speed ν and position ξ, as well as the acceleration jump for Case 1 depend on the value of the parameter Λ. By increasing Λ this moment comes earlier and the jump decreases in amplitude (see Eqs. 1, 2 and 3 in Table 2). The space ξ, speed ν and acceleration α in Case 2 have a smooth time variation with a maximum of ν for $\tau = 0.5$.

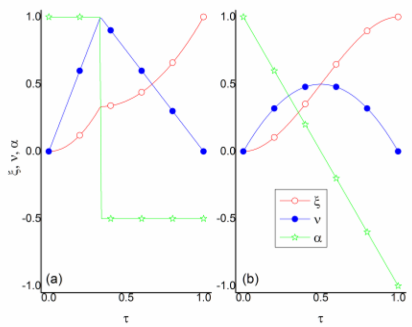

Figure 1. Dimensionless space ξ, speed ν and acceleration α as functions of dimensionless time, τ. (a) Case 1 for $\Lambda = 3$; (b) Case 2.

The calculation of the non-dimensional controlled potential energy, $\chi(\xi)$, was done differently in the two cases. For Case 1, Eq. 4 of Table 2 was obtained by space integrating of Eq. (21) in dimensionless form. In Case 2, the calculation of $\chi(\xi)$ of Eq. 8 from Table 2 was further elaborated. First, the function $\alpha(\xi)$ was calculated numerically, by extracting the variable τ from the function $\xi(\tau)$ (Eq. 5 of Table 2) and replacing it in the function $\alpha(\tau)$ (Eq. 7 of Table 2). Then the function $\alpha(\xi)$, obtained tabularly, was fitted with software TableCurve2D (TableCurve2D, 2002) by using the following equation:

$$\alpha(\xi)=\frac{a+c\xi^{0.5}+e\xi+g\xi^{1.5}}{1+b\xi^{0.5}+d\xi+f\xi^{1.5}} \tag{39}$$

where the values of the fitting coefficients are given in Table 3. It was then calculated tabularly $\chi(\xi)$ by using Eq. 8 of Table 2. Finally, the table of values $\chi(\xi)$ was fitted by using software (TableCurve2D, 2002), with the function:

$$\chi(\xi)=\frac{a+c\xi^{0.5}+e\xi+g\xi^{1.5}+i\xi^{2}+k\xi^{2.5}}{1+b\xi^{0.5}+d\xi+f\xi^{1.5}+h\xi^{2}+j\xi^{2.5}} \tag{40}$$

and the fitting coefficients may be found in Table 3.

Table 3. Values to be used for the coefficients of Eqs. (39) and (40).

Coefficient	Equation (39)	Equation (40)
a	0.9976	0.05848
b	-2.1379	-18.4237
c	-3.2580	-1.3817
d	1.3481	132.8090
e	3.4618	12.1266
f	-0.2099	-227.5588
g	-1.2016	-49.1109
h	-	180.2209
i	-	51.9175
j	-	-59.7951
k	-	-13.6987

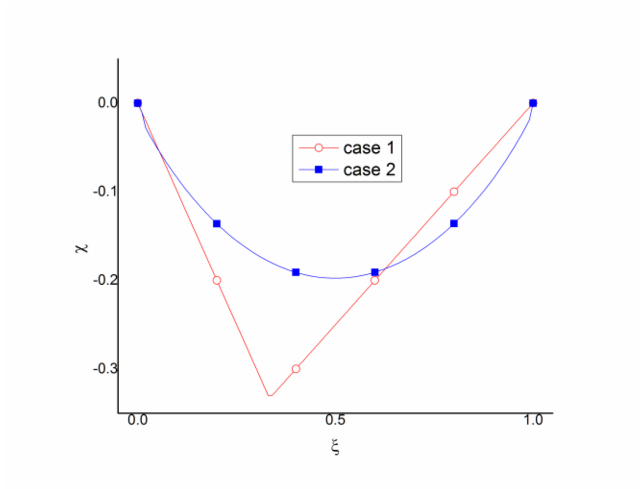

Figure 2. Dimensionless controlled potential energy $\chi(\xi)$ for Case 1 ($\Lambda = 3$) and Case 2

Figure 2 shows the controlled potential energy $\chi(\xi)$ for the two cases. In Case 1 the place of the minimum of $\chi(\xi)$ and its value depend on the value of parameter Λ. This place moves to the left when Λ increases. The minimum of $\chi(\xi)$ occurs at $\xi = 0.5$ in Case 2.

Notice that N is not an ordinary potential energy but a controlled potential energy. Indeed, N allows to obtaining not only the kinematics and dynamics of the particle at any intermediate time but also the initial and final states, including the initial and final shape of the system. Therefore, N is "holding its completion in itself", in good agreement with the properties of *entelecheia* as described by Aristotle.

The maximum speed v_{max} and the work L performed by the controlled force F_N are larger in Case 2 than in Case 1 (compare Eqs. (7) and (37), and Eqs. (13) and (38), respectively). Therefore, the same shape may be obtained, in the same amount of time, by using different amounts of work. This is in good agreement with the common perception that things having the same shape may be manufactured with different amount of resources (work and material).

Now we shall focus on the kinetic energy and controlled potential energy of the system. The maximum value of the kinetic energy T is given by:

$$T_{max} = \frac{mv_{max}^2}{2} \tag{41}$$

The dimensionless kinetic energy ε is defined as follows:

$$\varepsilon \equiv \frac{T}{T_{max}} = \left(\frac{v}{v_{max}}\right)^2 = \nu^2 \tag{42}$$

In Case 1 the time variation of the dimensionless kinetic energy $\varepsilon(\tau)$ is obtained by using Eq. (42) and Eq. 2 of Table 2 while in Case 2, $\varepsilon(\tau)$ is computed by using Eq. (42) and Eq. 6 of Table 2. If the time variation of $\chi(\tau)$ is wanted, a transform $\chi(\xi(\tau))$ is used, as follows. In Case 1, Eqs. 4 and 1 of Table 2 are used while for Case 2 one uses Eq. 5 of Table 2 and Eq. (40).

The interpretation of the convergence between *energeia* and *entelecheia* in terms of modern physics requires some comments. Convergence may be defined for two functions of the same parameter, whose values become closer and closer when the value of the parameter tends towards a certain limit. However, another kind of convergence criteria seems to be more appropriate in case of energy and entelechy. We may define two functionals, depending on all values taken during the motion by energy and entelechy, respectively. The values of the two functionals are real numbers and therefore they may be compared. One may say that the difference between energy and entelechy is larger or smaller, pending on these two values and this procedure may be used to define the convergence criterion between the two quantities.

Next, we remind that a common functional involving energy is action. The idea is that we may compare energy and entelechy in terms of their actions. Convergence between the two quantities may be interpreted as a minimization of the difference between their associated actions.

Metaphysical principles assuming the extremization of action have been adopted in physics starting with 18th century, with main contributions from Pierre-Louis Moreau de Maupertuis, Leonhard Euler and others. The perception on these principles changed in the next century, mainly because the generalization beyond the realm of mechanics, in particular in the realms of statistical physics and field theory, has not been covered by the initial metaphysical assumptions. Despite several distinguished scientists, such as Henri Poincaré or Fred Hoyle, were still reluctant in using these principles, the general consensus is that they are simply convenient tools to treat in a compact way the laws of physics (Veldman 2024).

One such tool is the Hamilton principle, which states that the evolution of the system is a stationary point of the action functional, which is defined as the time integral of the difference between kinetic energy and potential energy. That is the reason of using the Hamilton principle in our tentative to find a place in physics for entelechy, as a quantity which is converging toward energy.

The kinetic energy T and the controlled potential energy N may be used to build the Lagrangian Γ of the movement, which is defined as usual:

$$\Gamma \equiv T - N \tag{43}$$

The action A of the movement is defined as:

$$A \equiv \int_0^{t_f} \Gamma dt \tag{44}$$

In terms of the dimensionless quantities $\varepsilon(\tau)$ and $\chi(\tau)$, Eq. (44) turns into:

$$A = a' x_f \int_0^1 \left(\frac{2}{\Delta} \varepsilon - \chi \right) d\tau \tag{45}$$

$$A = \frac{9x_f^2}{t_f^2} \int_0^1 \left(\varepsilon - \frac{2}{3} \chi \right) d\tau \tag{46}$$

for Cases 1 and 2, respectively. Here Eq. (42) and the definitions of Table 1 for ν and χ have been used.

It is easy to show that the Hamilton principle yields Eqs. (21) and (18). Hamilton principle asks that for a real movement the action is stationary, i.e.

$$\delta A = 0 \tag{47}$$

But the action A has the form:

$$A = \int_{t_1}^{t_2} G(x, \dot{x}) dt \tag{48}$$

where the dot means time derivative and:

$$G(x, \dot{x}) \equiv \frac{m\dot{x}^2}{2} - N(x) \tag{49}$$

The following relationships apply:

$$\frac{\partial G}{\partial x} = -\frac{dN}{dx} \tag{50}$$

$$\frac{\partial G}{\partial \dot{x}} = m\dot{x} \tag{51}$$

The stationarity of the action A, Eq. (47), is ensured when the following Euler-Lagrange equation is fulfilled (Forray, 1968):

$$\frac{\partial G}{\partial x} - \frac{d}{dt}\left(\frac{\partial G}{\partial \dot{x}}\right) = 0 \tag{52}$$

Usage of Eqs. (50), (51), (52) and (21) yields:

$$m\ddot{x} = F_N \tag{53}$$

which is precisely Eq. (18) since $\ddot{x} \equiv a$.

Therefore, it may be said that the kinetic energy T and the controlled potential energy N converge, in the sense that the action associated with the difference between these two quantities is stationary ($\delta\int_0^{t_f}(T - N)dt = 0$). Or, this is one of the properties owned by the couple formed of *energeia* and *entelecheia*, as defined by Aristotle. Since we agreed that T has the properties of kinetic energy then we may conclude that the controlled potential energy N has the properties of entelechy.

Figure 3 shows the time variation of the dimensionless kinetic energy $\varepsilon(\tau)$ and dimensionless controlled potential energy (or entelechy) $\chi(\tau)$ which ensures the stationarity of Eqs. (45) and (46), for Case 1 and 2, respectively. The entelechy $\chi(\tau)$ is smaller in Case 1 than in Case 2 (Fig. 3b). In Case 1 the moment of the maximum of $\varepsilon(\tau)$ and minimum of $\chi(\tau)$ and their values depend on the value of parameter Λ. This moment moves to the left when Λ increases. The maximum of $\varepsilon(\tau)$ and the minimum of $\chi(\tau)$ occur for $\xi = 0.5$ in Case 2.

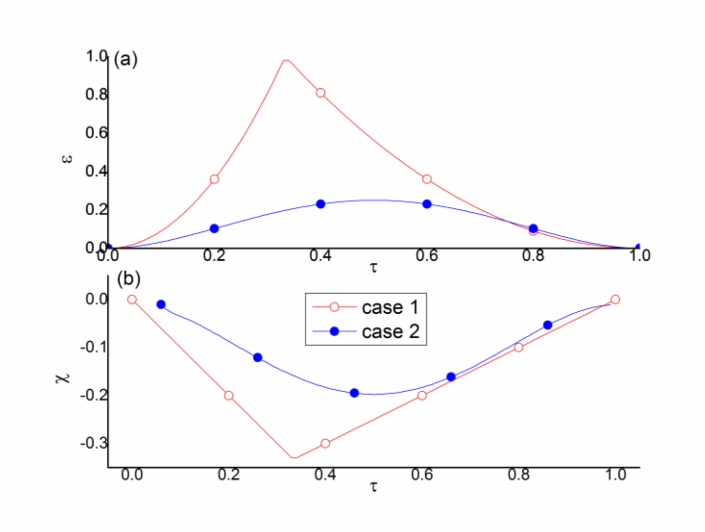

Figure 3. Dependence on the dimensionless time τ for (a) dimensionless kinetic energy $\varepsilon(\tau)$ and (b) dimensionless controlled potential energy (entelechy) $\chi(\tau)$, ensuring the stationarity of action defined by Eqs. (45) and (46) for Cases 1 ($\Lambda = 3$) and 2, respectively.

Notice that the Hamilton principle may be used as a criterion of convergence between kinetic energy and potential energy for any sort of motion, controlled or uncontrolled. Therefore, any future tentative to find the hypothetical entelechy of uncontrolled motions must prove that this quantity is "holding its completion in itself".

Conclusions

Aristotle coined two words, *energeia* and *entelecheia*, in his famous description of what change is. The meaning of these two words is still under debate but a convenient few-words translation may be as follows. *Energeia* may be translated as "being-at-work" while *entelecheia* means "holding its completion in itself". The two concepts have ontological coverage and may be used for any kind of particular change, including physical motion.

Modern physics adopted energy as one of its basic concepts, built on a modified form of Aristotle's *energeia*. Energy is a condition that describes the capacity to do work and may be seen as the equivalence class of the *energeia* of all things. Therefore, it is expected that *entelecheia* should also have a counterpart in modern physics. However,this requires a special investigation since we do not have yet an unbroken historical chain of connections leading from Aristotle's notions to the modern notions. The present work seems to be a first trial on the road.

We restricted here to controlled motions, which are better connected with the teleological character of Aristotle's definition of motion. We used a simple mechanical system and identified a quantity, called entelechy, which obeys the two properties of Aristotle's entelecheia*, i.e. (a) it is "holding its completion in itself" and (b) converges towards kinetic energy, which is the modern equivalent of* energeia*. The entelechy is a special sort of potential energy, namely a controlled potential energy.*

We have also found that the shape of the system may be controlled in different ways, characterized by different entelechies and different amounts of work, in good agreement with common perception.

The procedure to find the entelechy of a controlled motion is as follows. First, the kinematics is found from motion prescriptions. Second, the associated dynamics is determined. Third, the controlled potential energy making this dynamics possible is evaluated. This is entelechy, denoted N, which is a special case of potential energy that allows to obtaining not only the kinematics and dynamics of the system at any intermediate time but also the initial and final states, including the initial and final shape of the system.

Notice that a new class of control systems has been identified in case of non-holonomic mechanics (Soltakhanov et al., 2009) and this opens perspectives for the application of the theory for a larger class of systems. There is a need, of course, for further investigations.

Any future tentative to find the entelechy of uncontrolled motions must prove that this quantity is "holding its completion in itself" since the Hamilton principle may be used as a convergence criterion between kinetic energy and potential energy for any sort of motion, controlled or uncontrolled.

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Paper Received January 5, 2025; Accepted March 18, 2025; Published May 2, 2025